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Ministry of Education
Ontario

Hon. Bette Stephenson, M.D., Minister
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OAIP

ONTARIO
ASSESSMENT
INSTRUMENT
POOL

REPORT OF THE

1981 Field Trials in Mathematics

Intermediate Division

Leslie D. McLean

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Ministry
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Telephone 416/965- 6789 15th Floor
Mowat Block
Queen's Park
Toronto, Ontario
M7A 1L2

This report should be distributed to the those who received a copy of Mathematics, Intermediate Division, Ontario Assessment Instrument Pool, 1980. Teachers using the Mathematics pool will find the performance data in this report of interest. It should be noted that some teachers may wish to use the materials in this report as an alternative to the 1980 pool.

CONTRIBUTORS

Project Team

Principal Investigator	Les McLean
Research Officers	Vincent Gaudino
	Austin Elias
Secretariat	Joyce Townsend
	Terry Massameno
	JoAnne Squires
Computer Systems	Ron Ragsdale
	Richard Wolfe
	Norman Salt
Data Entry Supervisor	Kathy Larmour
Graphic Design	Sarah Palmer
Writing	Martha Schmidt

Mathematics Subject Matter Group

The committee below advised on the 1981 Field Trial. The final report, however, was the responsibility of the Principal Investigator and the Project Team.

David Alexander (Faculty of Education, University of Toronto)
David Aylsworth (Ontario Teachers' Federation)
Jim Doris (Ministry of Education)
Jim MacLean (Ministry of Education)
Alex Norrie (Peel Board of Education)
George Scroggie (Ministry of Education)
W. G. Anglin (Frontenac County Board of Education)
Sam Pravica (Faculty of Education, University of Western Ontario)

Subject Advisory Group

The committee below advised on the development of the published pool.

Don Fox (Ministry of Education, Eastern Ontario Regional Office)
Pen Baxter (Leeds and Grenville Board of Education, OTF Representative)
W. C. Higginson (Faculty of Education, Queen's University)
Nick Roehrig (Waterloo County RCSS Board, OTF Representative)
W. G. Anglin (Frontenac County Board of Education)
Roger Wilson (Toronto Board of Education, OTF Representative)
Michael Silbert (Hamilton Board of Education)
Jack Spohn (Centennial College of Applied Arts and Technology)

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A project as large as this one succeeds only with the co-operation of all parties. Primary thanks go to the students, teachers and administrators who received our materials, interpreted the instructions to their schools and returned the completed booklets in excellent condition.

Administrative support during the first and most difficult part of the project was provided by Vivienne Jesudian of OISE's Department of Measurement, Evaluation and Computer Applications. We owe much to her and to Chairman Ross Traub for their early support. During the latter part of the project, this support was provided by Amelia Nanni, Marion Harris, Angela Hildyard and others in the OISE Office of the Assistant Director, Field Services and Research.

Austin Elias, one of the two full-time Research Officers on the Project Team, deserves special mention. He did most of the detail work and contributed to the report.

This large, complex dataset could not have been entered to the computer, processed and analysed without the special efforts of staff in OISE's Computing Services Group. Tina Edwards, Jack Pledger, and others gave of themselves well beyond the call of duty. Thanks are also due the OISE Press for helping to get our 106 different booklets and four newsletters printed within difficult deadlines.

As noted, the Mathematics Subject Matter Group advised on the field trial. What should be added is that they were unfailingly helpful and tolerant as the project team slogged and sometimes staggered through this first ever field trial in Ontario. Jim Doris, the Ministry's Supervisory Official, a problem solver, was a valued and needed steady hand.

PREFACE

The more things change . . .

> The problem of teaching children the properties of integers and how to reckon with them, and of leading them on to complete mastery, is very difficult and requires the labour of several years, from the first school year until the child is ten or eleven years old.

the more they remain the same

> After [Grade 7], and especially after [Grade 8], arithmetic begins to take on the more aristocratic dress of mathematics . . . This represents such a long step in abstraction that one may well declare that real mathematics begins with operations with letters. Naturally, this transition must not be accomplished rapidly. The pupils must accustom themselves gradually to such marked abstraction.

Felix Klein
Gottingen, 1908

Dazzled by the scientific accomplishments of the twentieth century, we might overlook the more stately pace of evolution. In 1982, children are not so different from those observed by Felix Klein, the German mathematician and educator, at the turn of the century, and every teacher in elementary and secondary schools can identify with Klein's observations. They provide a useful backdrop for this report on the mathematics performance of eleven to sixteen year old students in Ontario in 1981.

The Ontario Assessment Instrument Pool

The Ontario Assessment Instrument Pool (OAIP) is a resource designed to assist educators in evaluating student achievement and the effectiveness of school programs. It is not intended, nor should it be used, for purposes of teacher evaluation.

OAIP materials have been developed jointly by classroom teachers, curriculum specialists, and experts in testing, to ensure the materials' practicality in the classroom as well as their quality as assessment tools. When the research contracts in progress (as this report is being written) have been completed, OAIP will consist of sixteen "pools" of materials, each one relating to a different curriculum area. The materials will correspond to the topics in the respective Ministry of Education curriculum guidelines and, as such, can provide assessment materials for use throughout the province.[1]

Each pool is composed of a collection of "instruments". An instrument may take any of a variety of forms; it may be a single question, or a written passage with accompanying questions, or, perhaps, a map with questions. From the supply of instruments available, teachers may select appropriate ones to administer to their students. The instruments may be used on their own or in combination with teachers' questions in the classroom. Most pools will have at the beginning a table that relates the instruments in the pool to the topics in the curriculum guideline. The organization scheme used in many of the pools is illustrated in Figure 1.

The 1980 Mathematics Pool

In 1980, the Ministry of Education published *OAIP Mathematics: Intermediate Division.* This 1980 edition covers some, but by no means all, of the mathematics curriculum in Grades 7 through 10. Several content areas, such as geometry, will be added at a later date. On the other hand, many topics covered in previous grades were included if students had to have covered them in order to move on to Intermediate Division mathematics. Since most of these prerequisites are reviewed in Grades 7 through 10, it was thought appropriate to include them in the pool. Results from the field trial confirmed that teaching of most prerequisites continues through the Intermediate Division.

The contents of the *published* pool are listed in Table 1. Each content area is made up of several topics which, in turn, include specific objectives. There is always a "terminal" objective, and there may be one or more "enabling" objectives. Enabling objectives are simpler versions of an objective; they usually keep to one operation or complication. The terminal objective then combines all aspects of the enabling objectives.

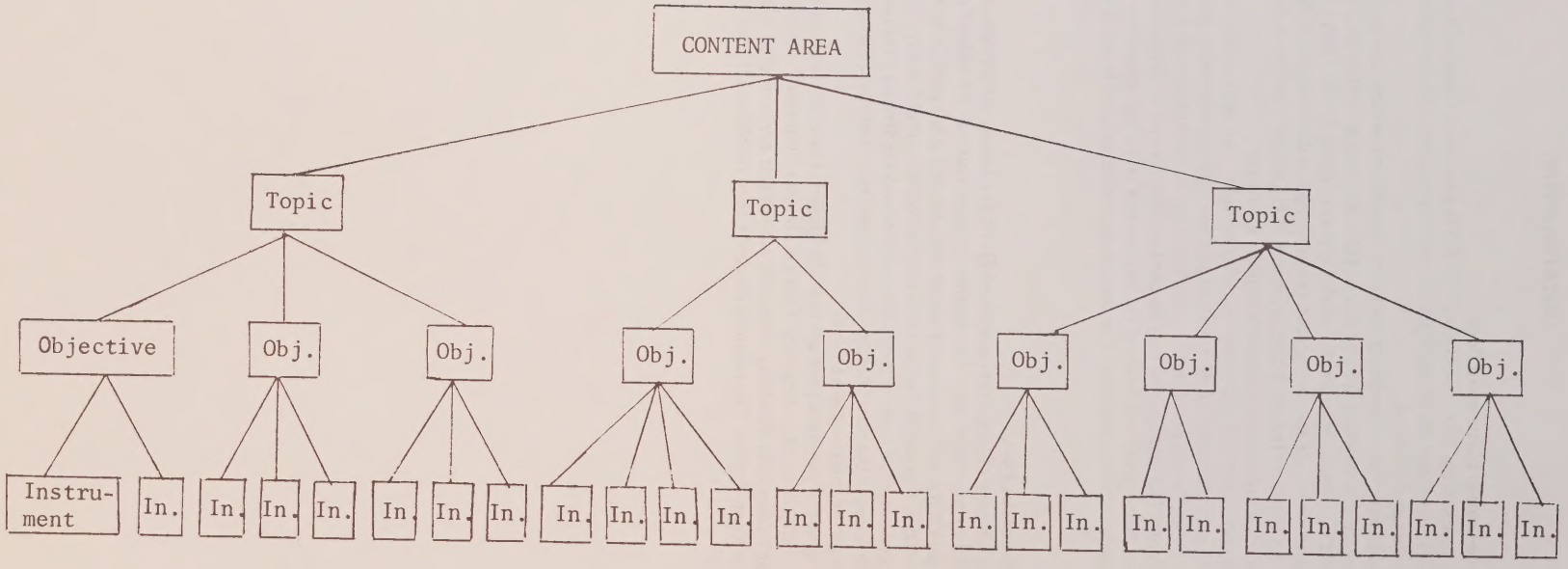


Fig. 1. Organization of the Instrument Pools

Table 1. Table of Contents for OAIP Mathematics Published Pool

NUMERICAL METHODS

N001	Addition of Whole Numbers
N002	Subtraction of Whole Numbers
N003	Multiplication of Whole Numbers
N004	Division of Whole Numbers
N005	Factoring
N006	Roots of Perfect Squares
N007	Exponents
N008	Order of Operations
N009	Addition and Subtraction of Decimals
N010	Multiplication of Decimals
N011	Division of Decimals
N012	Multiplication of Fractions
N013	Division of Fractions
N014	Addition and Subtraction of Fractions
N015	Conversion of Decimal Fractions to Common Fractions
N016	Converting Common Fractions to Decimal Fractions
N017	Two Term Ratios
N018	Expressing Per Cent in Different Ways
N019	Working with Per Cent
N020	Adding Integers
N021	Subtracting Integers
N022	Multiplying Integers
N023	Dividing Integers
N024	Multiplying Rational Numbers in Fraction Form
N025	Dividing Rational Numbers in Fraction Form
N026	Adding and Subtracting Rational Numbers in Fraction Form
N027	Adding and Subtracting Rationals in Decimal Form
N028	Multiplying and Dividing Rational Numbers in Decimal Form
N029	Applications
N030	Measurement of Plane Shapes
N031	Measurement of Three-Dimensional Shapes

ALGEBRA

A001	Introducing Variables
A002	Translating Simple Algebraic Expressions
A003	Evaluating Expressions
A004	Solving Equations by Inspection
A005	Solving Inequalities
A006	Translating of Complex Algebraic Expressions
A007	Working with Expressions
A008	Simplifying Expressions in One Variable
A009	Solving Simple First Degree Equations in One Variable
A010	Solving Complex First Degree Equations in One Variable
A011	Solving First Degree Equations in One Variable Involving Rational Expressions
A012	Using Formulae
A013	Learning About Probability
A014	Determining Probabilities in Simple Two-Outcome Experiments
A015	Determining Probabilities in Multi-Outcome Experiments
A016	Expressing Percentage Probabilities as Fractional Probabilities
A017	Determining Odds
A018	Classifying Events
A019	Using the Multiplication Law

Unlike the other pools, the 1980 Mathematics Pool does not contain specific instruments, but rather general formulas or "models" from which many different instruments can be generated. Following each model are several examples indicating how the model might be used. One of the algebra models is shown in Table 2.

Table 2. Example of Instrument Model from the Mathematics Pool

TOPIC A008000: Simplifying Expressions in One Variable

OBJECTIVE A008200: Removing brackets and collecting like terms (positive coefficients)

MODEL: Simplify: $a(by \pm c) + d(ey \pm f) = ?$
$y \in \{a,b,d,m,n\}$
$2 \leq a, d \leq 10$
$0 \leq b,c,e,f \leq 10$ $\quad b \neq e$

EXAMPLES:

A008201 Simplify: $2(8n + 4) + 6(2n - 10) = ?$
$28n - 52$

A008202 Simplify: $10(6d + 6) + 6(10d - 5) = ?$
$120d + 30$

A008203 Simplify: $4(-1) + 6(2m - 4) = ?$
$12m - 28$

Purposes

The last stage in the development process for the mathematics instruments published so far was the field trial in schools across Ontario. This collection of responses from a fully representative sample of Ontario students is useful in several ways. First, remaining problems with the instruments, if any, can be detected. Second, the level of difficulty of each instrument can be determined for each grade and level, and these results then communicated to teachers. Finally, the results provide a province-wide picture of student performance on the topics included in the 1980 version of the pool.

Results from a representative sample of students can also provide a benchmark against which performance in future years can be compared. Changes in society (the spread of calculators and computers, for example), in the curriculum, in the mathematical symbols, and in the emphasis given to topics make comparisons over time very difficult, but results from many instruments and a large sample of students can provide a reasonable base for comparisons.

Selection of Participants

The Mathematics Pool was given a field trial in 180 schools across Ontario. Figure 2 shows the location of these schools, each of which was asked to carry out instrument testing in one mathematics class at each grade level (7-10) in the school. Schools having only Grades 7 and 8 were asked to do testing in two mathematics classes at each grade level. Where the school had more than one eligible class in a grade, the research team selected a class at random. In all, over 26000 students participated in the field trial.

How the schools were chosen was very important. A simple random choice of schools would have underrepresented the students in big cities. They are in larger schools but there are fewer of them. Sampling one hundred high schools at random could leave out the North entirely; Ontario has a total of almost six hundred high schools, and relatively few of them are located in the North.

All publicly funded English-language schools were eligible to participate in the survey, except special schools such as those for the deaf and blind. Thus, most separate schools were eligible (since they are publicly supported to Grade 10), but private schools such as Trinity, Lakefield, Ridley or Bishop Strachan were not. However, schools with very small enrolments—fewer than eleven students in both Grades 7 and 8 and fewer than fifty in Grades 9 and 10—were not sampled because it was too costly to go to these schools for so few responses. Less than one per cent of pupils was excluded by this last decision.

To avoid underrepresenting large schools, and to avoid leaving out the North and making other mistakes that would result in a bad sample, schools were first grouped in various ways. Thus, school size was a grouping (stratifying) variable, as was geographic region. Finally, 180 schools were selected from about sixty boards across Ontario. A check confirmed a distribution representative of the province: about half the schools from the Toronto region and half from elsewhere.

Ideally, the mathematics sample should have contained 2250 students in each grade. In Grade 9, the intended breakdown was: Basic 325, General 925, and Advanced 1000 students. The Grade 10 sample should have been: Basic 275, General, 1000, and Advanced 975 students. In reality, the mathematics sample turned out as follows: Grade 7, 2502; and Grade 8, 2583. The Grade 9 sample was: Basic 350, General 1093, and Advanced 1439. The Grade 10 sample was: Basic 205, General 925, and Advanced 1248. The Grade 10 sample departed most from the ideal, but none of the figures are cause for concern.

In order to determine if the sample is truly representative, one must also see how many students from each grade answered each form. The target was 140 responses to each instrument at each grade. Although it was not possible to get 140 responses per grade for every instrument, very few instruments received less than 130 responses per grade. In general, there were not enough responses to instruments from classes at Grade 9 and 10 Basic levels to give a precise estimate of performance at these levels. This outcome was inevitable, given the small number of Basic level classes. Moreover, students in Basic level classes omitted more questions than did other students.

The sample was sufficiently large and well distributed for accurate, precise statistics to be calculated by grade and by level for General and Advanced in 9 and 10.

Fig. 2. Schools Participating in 1981 OAIP Field Trials

Preparation and Administration

A number of schemes were considered for choosing the actual instrument sample for the field testing, since the pool itself contains instrument forms or models rather than instruments (see Table 2). It was decided to use a computer program to generate the specific instruments for each objective—six instruments for each terminal objective and four instruments for each enabling objective. This procedure established over 600 instruments to be field tested.

The sample of instruments having been established, a functional scheme was devised for administering these instruments to the student sample. No student could be asked to attempt all instruments, nor should all the algebra items be assigned to one booklet or be attempted by one class. Therefore, about thirty instruments were assigned to each booklet in such a way as to ensure (1) that every instrument in the sample was tested; (2) that it was tried out by a sufficient number of students to do a reliable analysis; (3) that students from different classes and different schools tried a given instrument; and (4) that any given student attempted instruments from a wide variety of topics. The large number of instruments and the process of assignment to booklets resulted in thirty different booklets, so that in almost every class each student was asked to complete a different test booklet. Instruments were randomly ordered within each booklet to reduce the instrument interactions and to make sure no objectives were always tested first or last in a booklet.

An "Opportunity to Learn" question was placed to the right of each instrument. By completing this statement, students indicated whether the information needed to answer the test instrument had been taught before this year, taught this year, or not yet taught. The occasion of the field trial was also used to survey student attitudes to mathematics and to gather information from teachers about themselves, about the way they approach their teaching, and about teaching resources.

In order to have results as comparable as possible across the province, teachers were asked not to permit the students to use calculators in completing the field trial booklets.D1U On the survey questionnaire, students were asked about calculator use. We now know that calculators are little used in Grade 7 and 8 classes (almost never on tests) but that usage increases greatly in Grade 9. Many students still did not make regular use of calculators, however, or at least so they reported in May 1981.

Practice varies widely from Basic and General (about 55 per cent use in class) to Advanced (90 per cent), and fully two-thirds of Grade 9 Advanced students report that they normally use calculators on tests. Percentages are lower in all streams in Grade 10. Interestingly enough, greater use was reported by students who are not going on to university, that is, those who are going to complete Grade 12 but not Grade 13.

Thus, the increasingly available hand-held calculator was a complicating element, even though students were requested not to use one in this field trial. It will not be possible to use these instruments in a future trial and obtain results under similar conditions. The use of calculators (and computers) will grow so rapidly in the next few years that the preferred course will probably be to permit students to use calculators and to ensure access to a calculator for every student. New instruments will have to be developed with calculators in mind, and it will be interesting to see how teachers incorporate the machines into their classroom instruction.

Reports and Materials

This report summarizes the performance results for terminal objectives (topics) and for groups of topics. It is not intended to be an exhaustive record of the OAIP Mathematics Field Trial.

A great deal of the detailed background information on the methods used in the field trial is contained in the four issues (February 1981 to May 1982) of the *Ontario Assessment Newsletter*, published by the OISE Educational Evaluation Centre. Much of the information on such aspects as sample design, administration, scoring, and analysis is included in the newsletters and is reproduced here as Appendix D.

Copies of the booklets used in the field trial (including the survey forms, there were forty-one different math booklets), the student responses and detailed summaries of the responses to each question have been transferred to microfilm. Copies of the film are available for study in the OISE library or may be purchased from the OISE Educational Evaluation Centre, Eighth Floor, 252 Bloor Street West, Toronto, M5S 1V6. Computer access to scores is provided by the Ontario Educational Data Archive, Computing Services Group, OISE. Address queries to the OEDA Archivist.

The student and teacher surveys are published as separate documents: *Willing but Not Enthusiastic: Student Attitudes to Mathematics in Grades Seven to Ten* and *Classroom Processes in Teaching Mathematics.* These reports are available from the Ontario Government Bookstore and OISE Publications Sales.

Processing the Data

As one might imagine, the open-endedness of the instruments led to a great variety of responses. A number of instruments produced 250 or more different responses, though many of these differences were insignificant. The result was an estimated total of 6300 different responses to be scored for the more than 600 instruments tested—quite an extensive job no matter how it is approached!

A correct answer was first determined for each instrument, and then responses that were mathematically equivalent or were the correct results of alternative valid solutions were scored as correct. If the question specifically asked for the answer to be rounded to two decimal places, however, only this version of the answer was scored as correct. If an instrument required specification of units in the answer (grams, metres, etc.) and no units were given, the response was considered incorrect, whether or not the number in the answer was correct. At this point, all scoring was done by one person. The computer was only used to gather and sort the responses. After the computer processing, the scorer had one list of all distinct responses to each instrument.

If the student had made no response at all, this was recorded as an omitted response. In calculating the statistical results, omitted responses were treated as incorrect, but the percentage of students failing to answer was reported separately as a statistic potentially interesting in itself.

A set of guidelines was devised and applied, but the scoring still required the application of reasonable judgement. All responses to several of the forms were also scored by two experienced math educators, with no significant variation in the results.

Presenting the Results

Ontario is a large province with a great deal of variety, and this factor is reflected in student performance. The province's diversity is difficult to capture and communicate. A single number would be a gross oversimplification (Ontario scores 70 per cent in algebra!), nor would one number per grade suffice. Given that students are taught together in classes, the average score in each class was computed. Displays were then prepared showing these averages for each grade, 7 through 10. For purposes of reporting, the mathematics topics were collapsed into eighteen groups (see Table 3), and a set of displays presented for each of these topic groups.

Table 3. OAIP Content Areas, Topic Groups and Topics for Which Displays of Student Performance are Provided on this Report

ARITHMETIC

Topic Group for Class Averages	Topics (Terminal Objectives), Each Represented in the Field Trial by Six Examples
1. Operations with whole numbers	Addition of Whole Numbers Subtraction of Whole Numbers Multiplication of Whole Numbers Division of Whole Numbers
2. Factoring and exponents	Factoring Roots of Perfect Squares Exponents
3. Order of operations	Order of Operations
4. Operations with decimals	Addition and Subtraction of Decimal Numbers Multiplication of Decimal Numbers Division of Decimal Numbers
5. Operations with fractions	Multiplication of Fractions Division of Fractions Addition and Subtraction of Fractions
6. Relating fractions with decimals	Conversion of Decimal Fractions to Common Fractions Conversion of Common Fractions to Decimal Fractions
7. Ratios	Two-term Ratios
8. Percentages	Per Cent Expressed in Different Ways (Ratio Approach) Working with Per Cent

Table 3 (continued)

APPLICATIONS	
9. Practical problems with whole numbers	Simple Whole Number Applications Whole Number Applications
10. Practical problems with fractions and decimals	Simple Fractions and Decimal Applications Fractions and Decimal Applications
INTEGERS AND RATIONALS	
11. Operations with integers	Addition of Integers Subtraction of integers Multiplication of Integers Division of Integers
12. Operations with positive and negative numbers in fractional form	Multiplying Rational Numbers in Fraction Form Division of Rational Numbers in Fraction Form Addition and Subtraction of Rational Numbers in Fractional Form
13. Operations with positive and negative numbers in decimal form	Addition and Subtraction of Rational Numbers in Decimal Form Multiplication and Division of Rational Numbers in Decimal Form
MEASUREMENT	
14. Basic measurement in two dimensions	Identifying Shapes Perimeter Basic Area
15. Measurement of trapezoid and circle	Trapezoid Circle
16. Measurement in three dimensions	Identifying Three-dimensional Shapes Volume
ALGEBRA	
17. Working with formulas	Translating Words to Mathematics Substituting Substituting in Formulas Simplifying
18. Equations and inequalities	Solving Linear Equations in One Variable Solving Equations (Simplification) Solving Equations (Rational Expressions) Inequalities Manipulating Formulas

In summary, each set of displays contains:

(1) for each terminal objective—a list of the six instruments tested for that objective;

(2) for each terminal objective—a display showing the range of per cent of correct responses to the six instruments, and for each grade (and level for 9 and 10);

(3) for the topic group—a display showing the range of *classroom means* on all the terminal objectives in that group, taken as a whole;

(4) for each display—preliminary interpretations provided by the members of the Mathematics Subject Matter Group and the Subject Advisory Group. (See page 3.)

Some sets contain additional material, which may be one or both of the following:

(1) a display showing the range of correct responses to instruments relating to *enabling* objectives;

(2) a display showing the relationship between students' achievement in a topic group and their *opportunity to learn* that material.

The following pages provide instructions for reading the various displays.

Reading the Display: "Range of Average Per Cent Correct Over All Classes on the Six Instruments in the Field Trial Measuring Achievement on One Terminal Objective" (Fig. 3)

1. For each grade level, the range of correct responses is indicated by two dots and by a horizontal line.
2. The dot to the left of the horizontal line indicates the instrument that was most difficult for the students. The dot to the right of the horizontal line indicates the easiest instrument.
3. The horizontal line between the dots represents the range of correct responses for the remaining four instruments.
4. Each horizontal has three vertical slashes. The slash at the left end of the bar represents the most difficult of these four instruments, and the slash at the right, the easiest. The slash in the middle of the bar indicates the average per cent correct for the four instruments.
5. Above the horizontal are the numbers of the six instruments, indicating the position of each instrument within the range. (These numbers were not included when the range covered less than twenty percentage points, because distinctions between relative positions would not be very useful.)
6. The heavy stippled line divides the Basic level results from the rest. As noted before, the sample size for the Basic levels was sufficiently small so that the results were not very reliable. The information on Basic classes should, therefore, be used cautiously, as a rough indication at best.

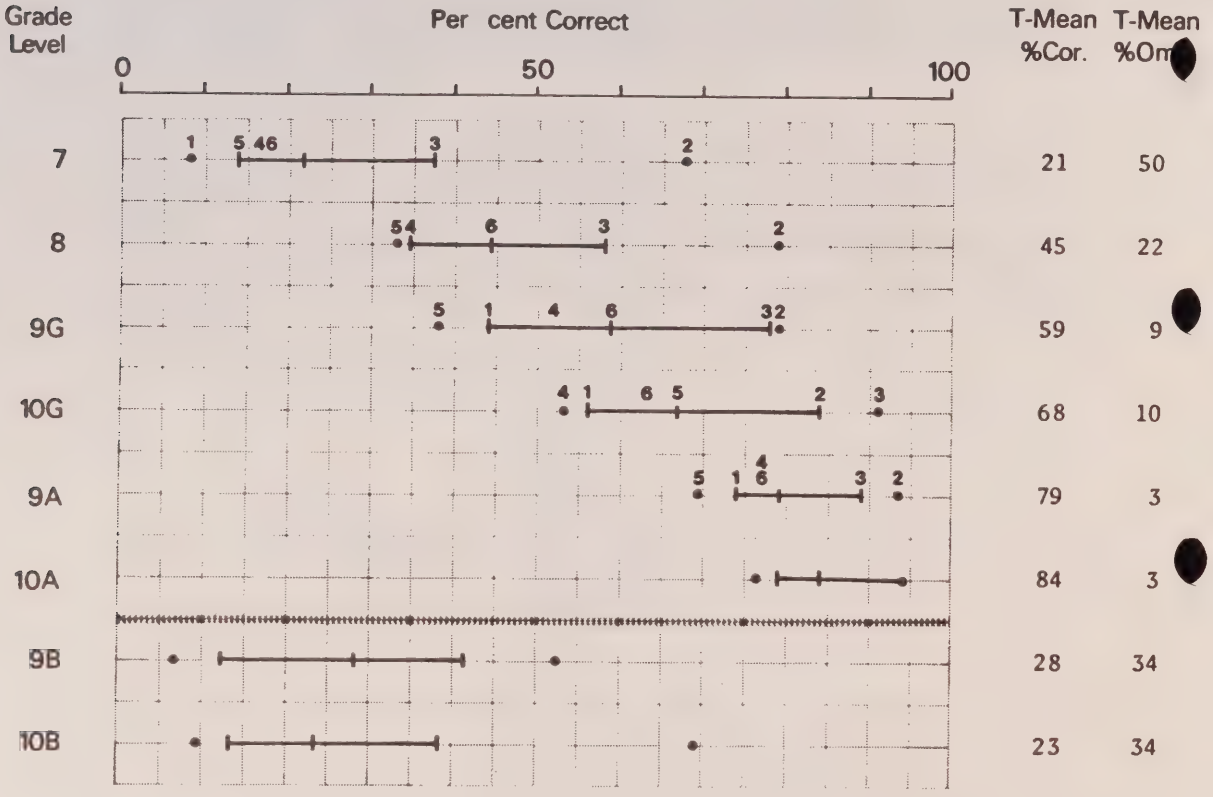


Fig. 3. Examples of Displays: "Range of Average Per Cent Correct Over All Classes on the Six Instruments in the Field Trial Measuring Achievement on One Terminal Objective".

7. At the right of the display is a column labelled "T-Mean, %Cor." (The "T" stands for trimmed.) This column contains the average per cent of correct responses to the four remaining instruments after the easiest and the most difficult were dropped (as mentioned in point 4 above). Trimmed means were chosen because they reflect a typical response rate more faithfully than if the extreme (and often unstable) results are included.

8. The second column, labelled "T-Mean, %Omits", contains the same sort of trimmed averages. In this case, the numbers represent the average per cent of omissions (non-responses). Again, the percentages are calculated by discarding the results from easiest and most difficult instruments.

9. Look at Figure 3, grade level 9A (Advanced). The most difficult instrument for these students was number 5; slightly less than 70 per cent of the students gave correct responses. The easiest instrument for these students was number 2 with approximately 94 per cent of the students answering correctly. Of the remaining, number 1 was more difficult than numbers 4 and 6, and number 3 was more difficult than number 2. The trimmed mean for the four non-extreme instruments was 79 per cent, and 3 per cent of the students gave no response at all.

In Grades 7 and 8, T-means are based on 500-600 responses. Grades 9 and 10 General and Advanced level T-means are based on 250-300 responses. The Basic level T-means were calculated from 80-100 student responses.

Reading the Display: "Range of Class Averages on a Topic Group, by Grade" (Fig. 4)

1. This type of display is referred to as a "box-and-whisker plot", the whiskers being the horizontal lines extending to the left and to the right of the box. A box-and-whisker plot is presented for each grade (all Grade 9s or Grade 10s together, whether Basic, General or Advanced).

2. The box covers the middle 50 per cent of the classes. The whiskers indicate the remaining 50 per cent of the classes: 25 per cent are to the left of the box and 25 per cent to the right. The whiskers show how far these two quarters extend. The cross (+) in the box marks the median or centre; half the classes in the grade scored better than the median value and half scored worse.

3. When a single class average is much smaller or larger than the rest (more than one and one-half box lengths to the left or to the right of the box), it is marked separately by an asterisk (*). A class average that is two or more box lengths away is indicated by a zero.

4. The box-and-whisker display was chosen for two reasons: it shows the wide range of student performance observed within grades, and it directs attention to the stable middle. If an entirely different sample of schools were chosen, the whiskers might be longer or shorter, but the boxes would very likely be the same.

5. Look at Figure 4. You will notice that (a) the class averages for the eighty-four Grade 7 classes cover a very wide range—7.5 per cent to 94.5 per cent; (b) the lowest twenty-one classes (the bottom 25 per cent) recorded averages between 7.5 per cent and 37.5 per cent; (c) the middle forty-two classes had averages between 37.5 per cent and 61.5 per cent; and (d) the upper twenty-one classes (the upper 25 per cent) averaged between 61.5 per cent and 94.5 per cent. The median was 45 per cent; that is, half the eighty-four classes were below this average and half were above it.

In Grade 8, one of the class averages was quite a bit lower than the rest (one and one-half box lengths). This single case is indicated by the asterisk. A similar case occurs in Grade 9.

6. *Statistical tests.* The sample statistics (found below each set of boxes-and-whiskers) may be used directly in statistical tests. The "Grade St. Dev." is based on variation among class means and hence contains variations due to the complex sampling design as well as variations in class size and number of items tested within each class. Taking the sample size as number of classes (rather than number of students) is also appropriate, though conservative. In Figure 4, the pooled std. dev. is 16.09. Thus, the difference between Grade 8 and Grade 9 is not significant ($t = 1.7$) but the difference between Grade 9 and Grade 10 is significant ($t = 3.1$, $p < .01$). A table of standard errors for the various displays is provided in Appendix E.

Reading the Display: "Achievement and Opportunity to Learn" (Fig. 5)

1. Student responses to the question "Taught—Before, This Year, or Not Yet" were retrieved for each group of topics and linked to their achievement. A scattergram was then produced, as seen in Figure 5. Across the bottom is a scale, "Opportunity to Learn, (OTL)", that ranges from zero (not yet taught) to ten (taught this year) to twenty (taught before). On the vertical scale to the left is the range of class averages, with the same per cent correct as in displays such as Figure 4.

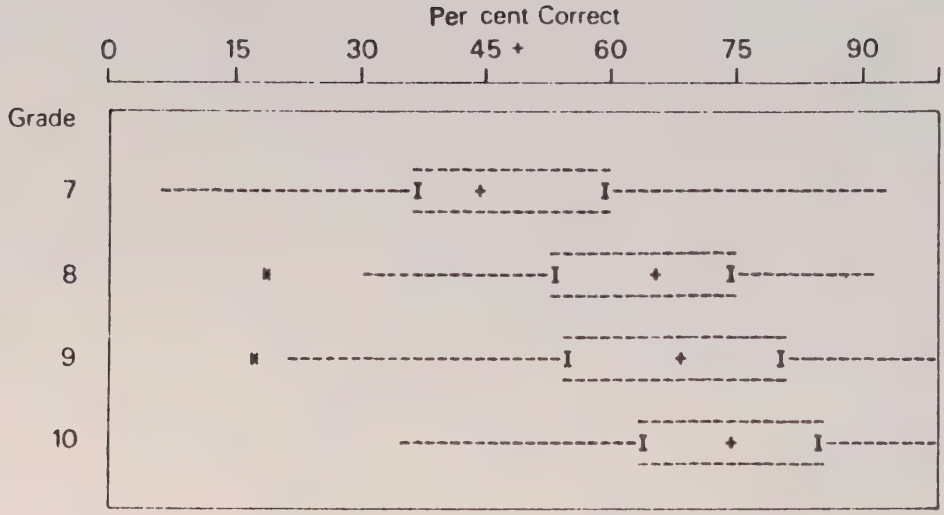
2. Each letter in the scattergram represents one class. Placement of a class on the scattergram is at the intersection of two lines—a vertical line at the class's average OTL on the bottom scale and a horizontal line at the class's average achievement on the scale at the left.

3. In Figure 5, most Grade 7 classes (letter G) are in the lower left corner. This indicates that students reported they had not yet been taught the material and also that their average score on this set of topics was low. The upper right corner is dominated by the Grade 9 and 10 classes (letters I and J)—indicating that students reported that the material was taught before this year and that their average score was quite high (some almost 100 per cent).

4. Most of the scattergrams looked like the example in Figure 5; higher achievement was linked to student reports that they had been taught the material, i.e., that they had an opportunity to learn. Low achievement was most often linked to low opportunity to learn. Student reports tended to agree with guideline suggestions for teaching each topic, lending credence to the reports. A few scattergrams deviated from the pattern, and these are reproduced later in this report.

Other Displays

Three of the objectives in the pool had instruments composed of two parts. Two of these objectives were in Measurement and the third objective was in Algebra. A slightly different method was used to score, code, and analyse the results from these items. The results are summarized in tabular form. The percentages of correct and incorrect responses are given separately for parts (a) and (b).



Statistical Summary

Grade Level	No. of Classes	Grade Mean	Grade St. Dev.
7	84	48.8	16.5
8	91	63.8	13.8
9	91	67.8	18.7
10	75	75.3	14.7

Fig. 4. Example of Displays: "Range of Class Averages on a Topic Group, by Grade".

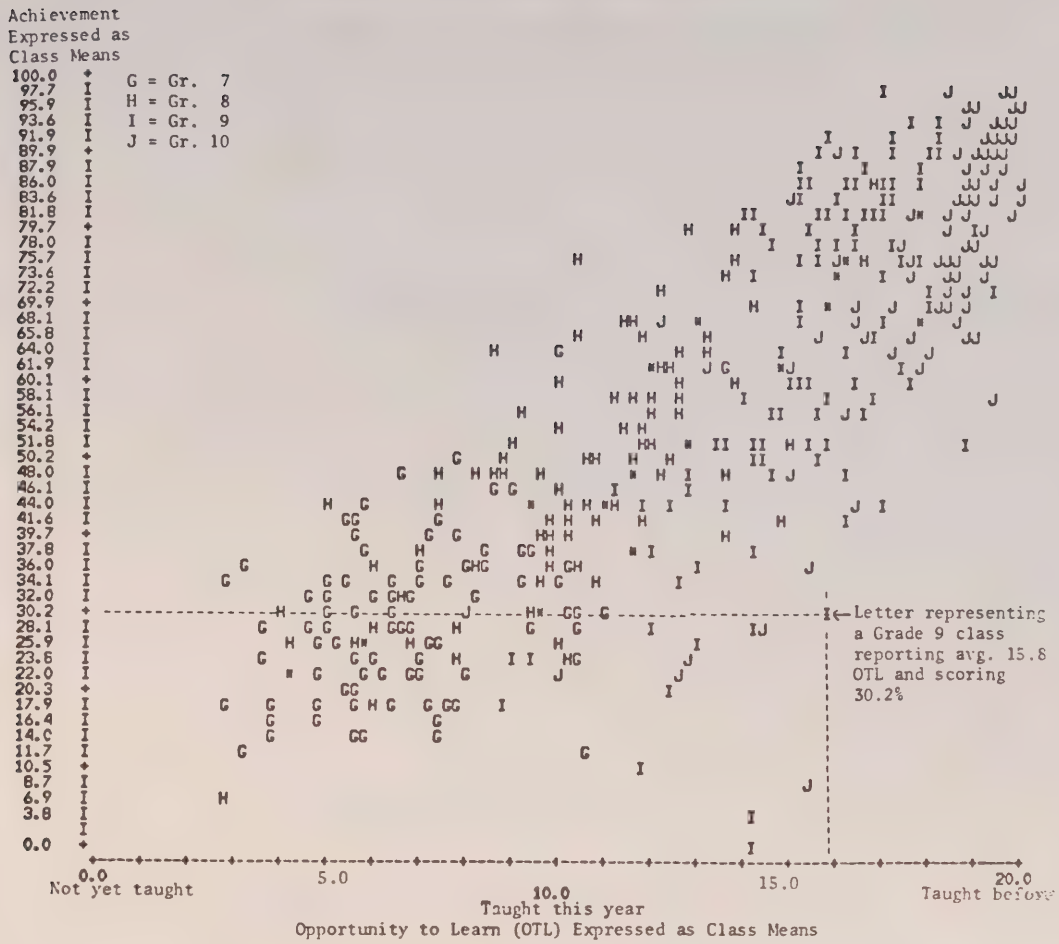


Fig. 5. Example of scattergram showing relationship between achievement and opportunity to learn. Each letter represents one class. Each asterisk (*) represents two or more classes.

OPERATIONS WITH WHOLE NUMBERS

Objective: Adding Four Numbers of Up to Four Digits (N 001 700; p. 7)

Instruments Included in the Field Trial:

(1) 5 + 8665 + 48 + 5084 =

(2) 326 + 81 + 7858 + 6 =

(3) 4835 + 3621 + 621 + 4655 =

(4) 3 + 2112 + 29 + 8371 =

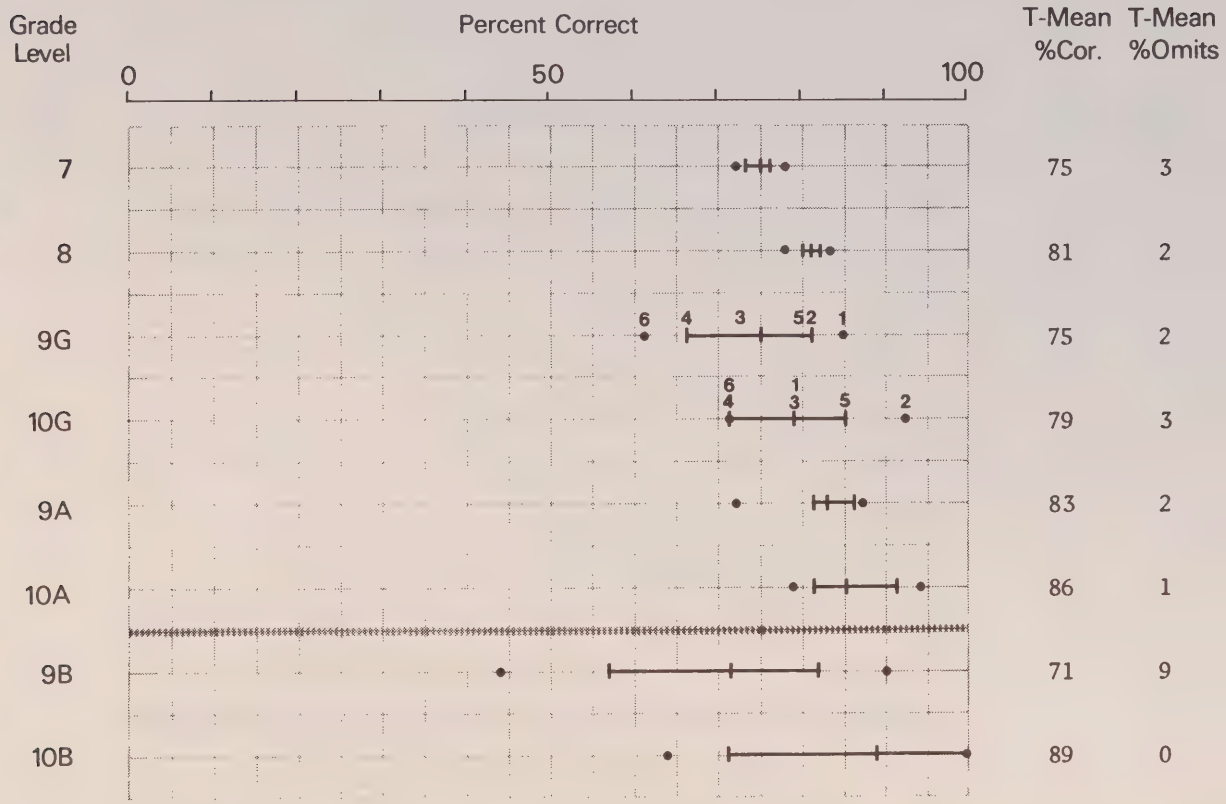
(5) 684 + 60 + 1745 + 7 =

(6) 9647 + 7924 + 404 + 4722 =

OPERATIONS WITH WHOLE NUMBERS

Objective: Adding Four Numbers of up to 4 Digits (N 001 700; p. 7)

Range of Correct Responses to the Six Instruments, by Grade:



- there is a high level of performance in Gr. 7 that is maintained and improved through Gr. 10.
- the wide range of performance on instruments by students in the basic level program may be accounted for by the small number of students tested.

Objective: Subtracting with Numbers of Up to Three Digits

(N 002 500; p. 13)

Instruments Included in the Field Trial:

(1) 866 - 64 =

(2) 382 - 156 =

(3) 740 - 116 =

(4) 173 - 32 =

(5) 623 - 186 =

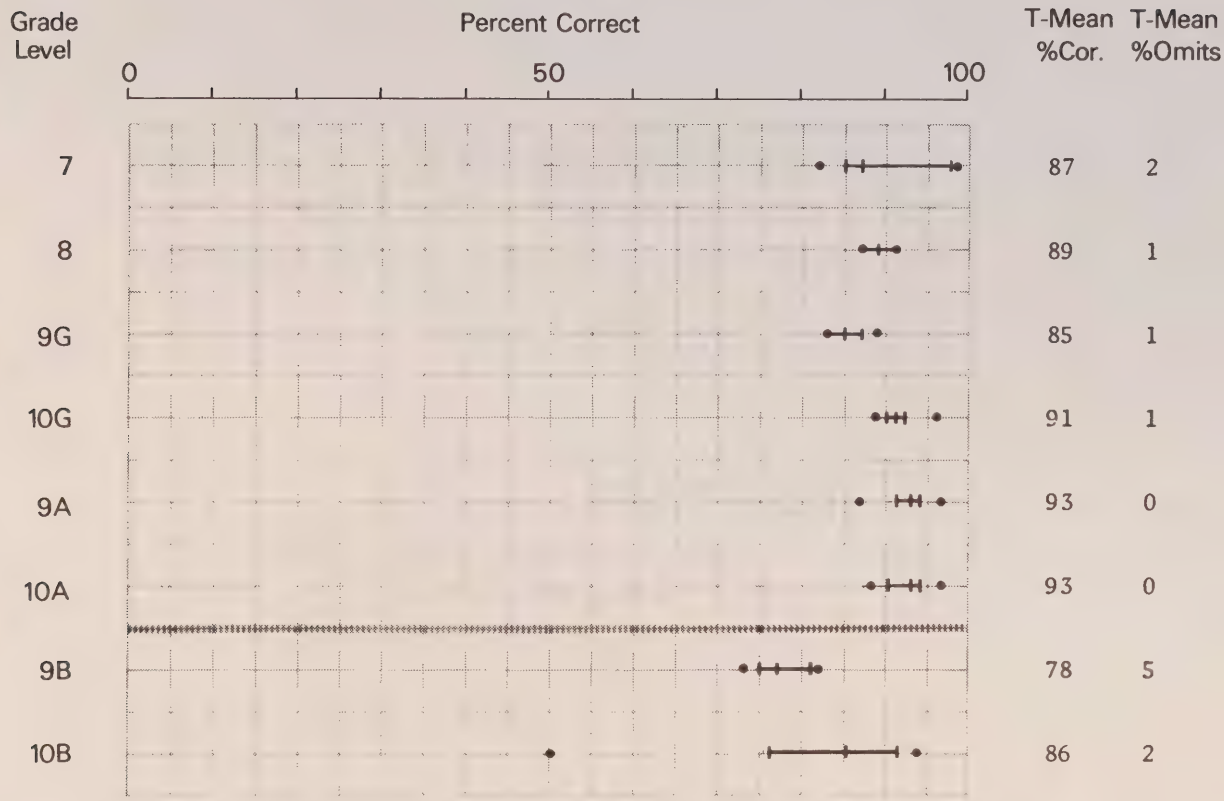
(6) 590 - 23 =

OPERATIONS WITH WHOLE NUMBERS

Objective: Subtracting with Numbers of Up to Three Digits

(N 002 500; p. 13)

Range of Correct Responses to the Six Instruments, by Grade:



- there is a reassuring level of success at all levels.
- results indicate that it is unnecessary to emphasize the subtraction algorithm in teaching most Intermediate Division students.

OPERATIONS WITH WHOLE NUMBERS

Objective: Multiplying Numbers of Up to Three Digits (N 003 700; p. 21)

Instruments Included in the Field Trial:

(1) $320 \times 708 =$

(2) $444 \times 97 =$

(3) $796 \times 999 =$

(4) $96 \times 297 =$

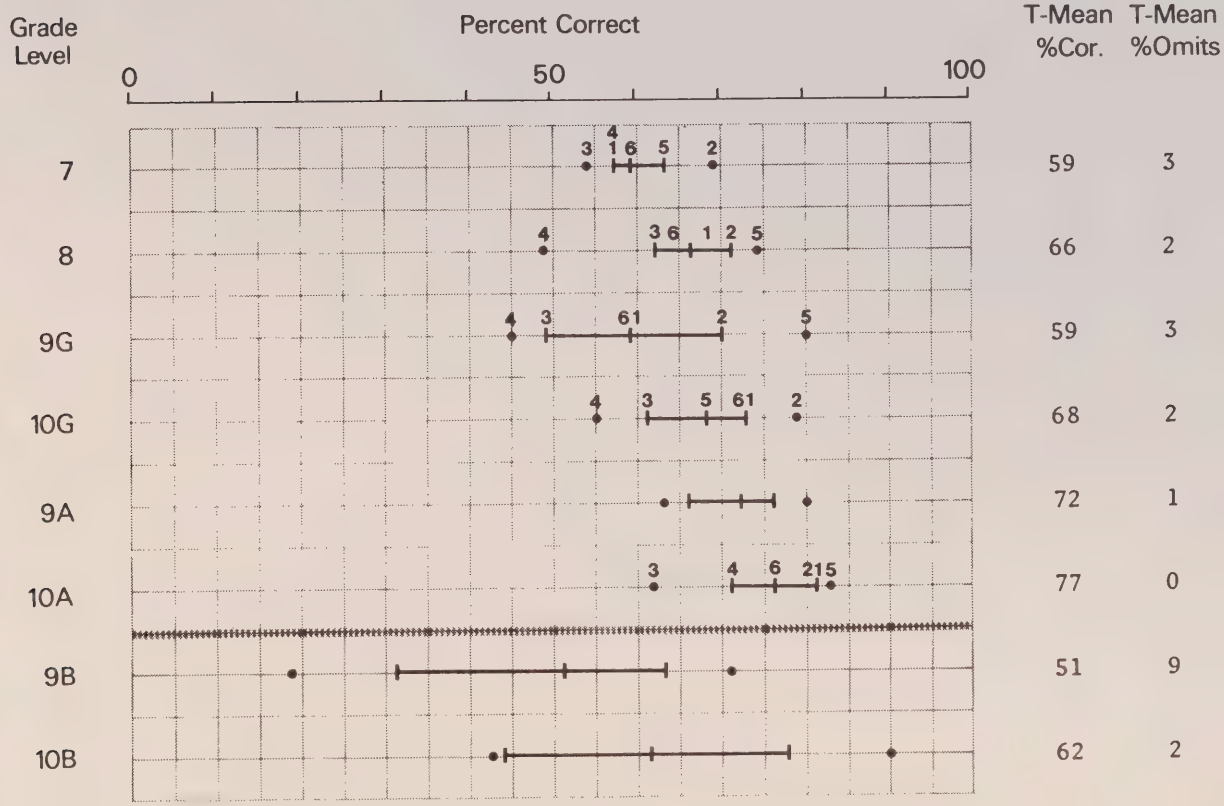
(5) $140 \times 702 =$

(6) $888 \times 17 =$

OPERATIONS WITH WHOLE NUMBERS

Objective: Multiplying Numbers of Up to Three Digits (N 003 700; p. 21)

Range of Correct Responses to the Six Instruments, by Grade:



- there is improvement in this skill across all grade levels, but results show that students find multiplication more difficult than addition and subtraction.
- students should have continued opportunities throughout the Intermediate Division to use the multiplication algorithm in practical situations.

Objective: Long Division (N 004 600; p. 28)

Instruments Included in the Field Trial:

(1) $27\overline{)407}$ =

(2) $13\overline{)290}$ =

(3) $37\overline{)415}$ =

(4) $11\overline{)111}$ =

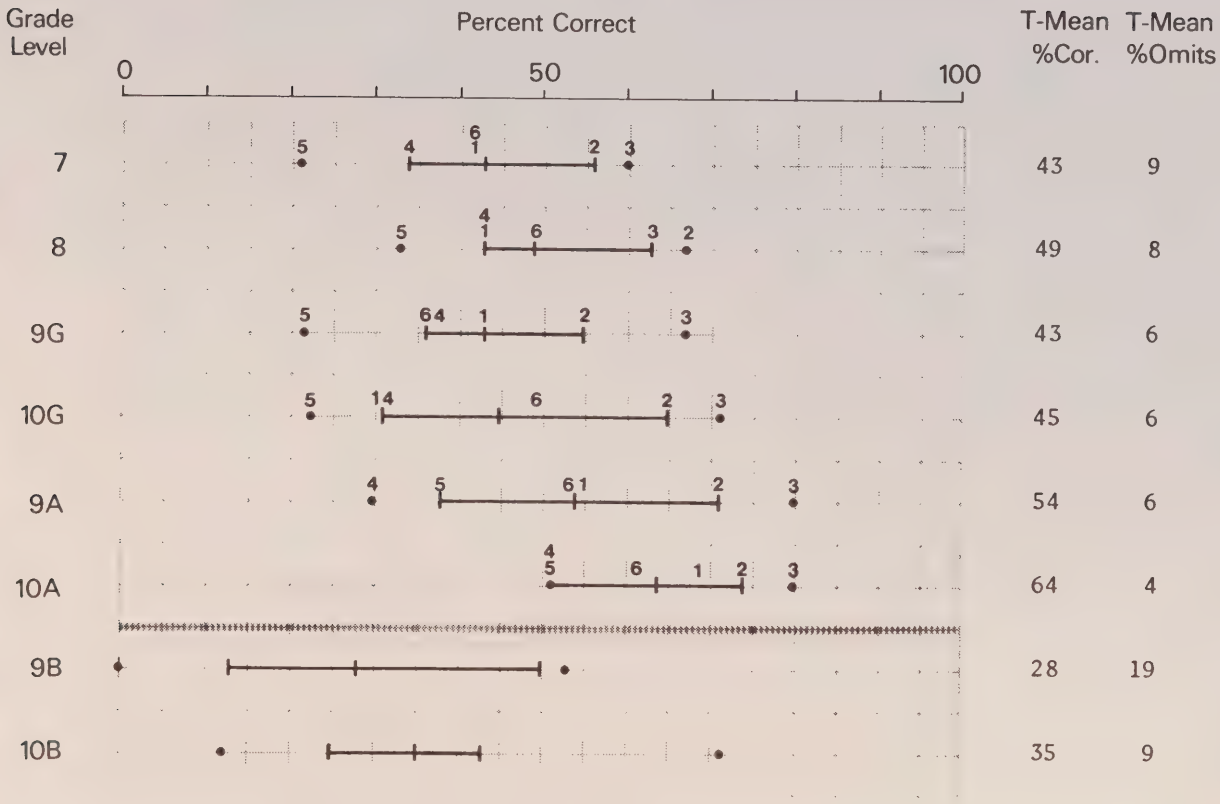
(5) $27\overline{)245}$ =

(6) $23\overline{)882}$ =

OPERATIONS WITH WHOLE NUMBERS

Objective: Long Division (N 004 600; p. 28)

Range of Correct Responses to the Six Instruments, by Grade:



- performance level is not as high as with the other operations because long division requires skill in both multiplication and subtraction.
- errors on any particular item do not necessarily reveal misunderstanding of the division algorithm. The mistake could be in the multiplication or subtraction operation.
- the range of performance could be partially explained by the difficulty students have dividing when a two-digit divisor yields a quotient with a large number; eg. 8 or 9.
- skills in multiplication and subtraction could be maintained by additional practice with division.
- technological developments in cash registers, calculators, etc. have reduced the demand for these skills.

OPERATIONS WITH WHOLE NUMBERS

Terminal Objectives in Topic Group

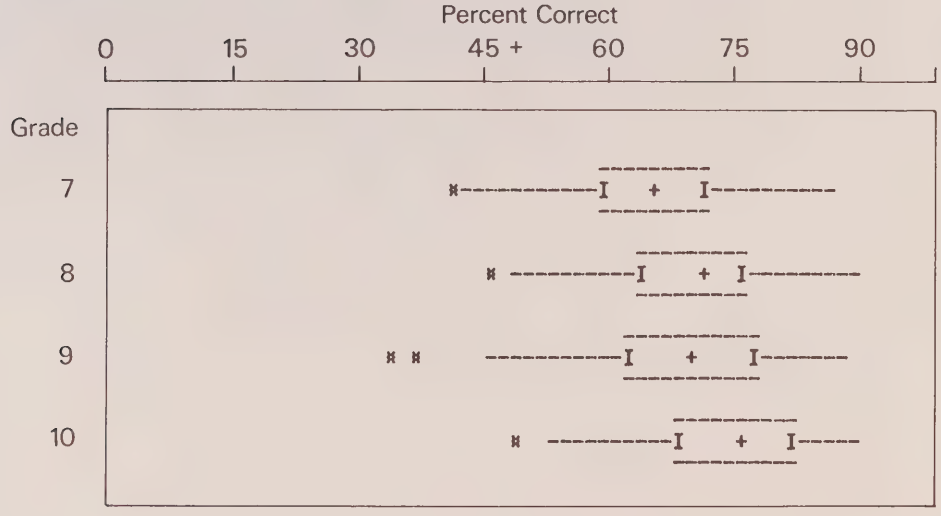
Adding Four Numbers of Up to Four Digits (N 001 700; p. 7)

Subtracting with Numbers of Up to Three Digits (N 002 500; p. 13)

Multiplying Numbers of Up to Three Digits (N 003 700; p. 21)

Long Division (N 004 600; p. 28)

Range of Correct Responses to Topic Group, by Grade:



Statistical Summary

Grade Level	No. of Classes	Grade Mean	Grade St. Dev.
7	90	66.0	8.9
8	94	71.5	9.2
9	102	70.2	10.7
10	89	75.9	9.3

Objective: Factoring a Number into Prime Factors (N 005 400; p. 35)

Instruments Included in the Field Trial:

(1) Write the following number as the product of its prime factors

30 =

(2) Write the following number as the product of its prime factors

54 =

(3) Write the following number as the product of its prime factors

36 =

(4) Write the following number as the product of its prime factors

110 =

(5) Write the following number as the product of its prime factors

175 =

(6) Write the following number as the product of its prime factors

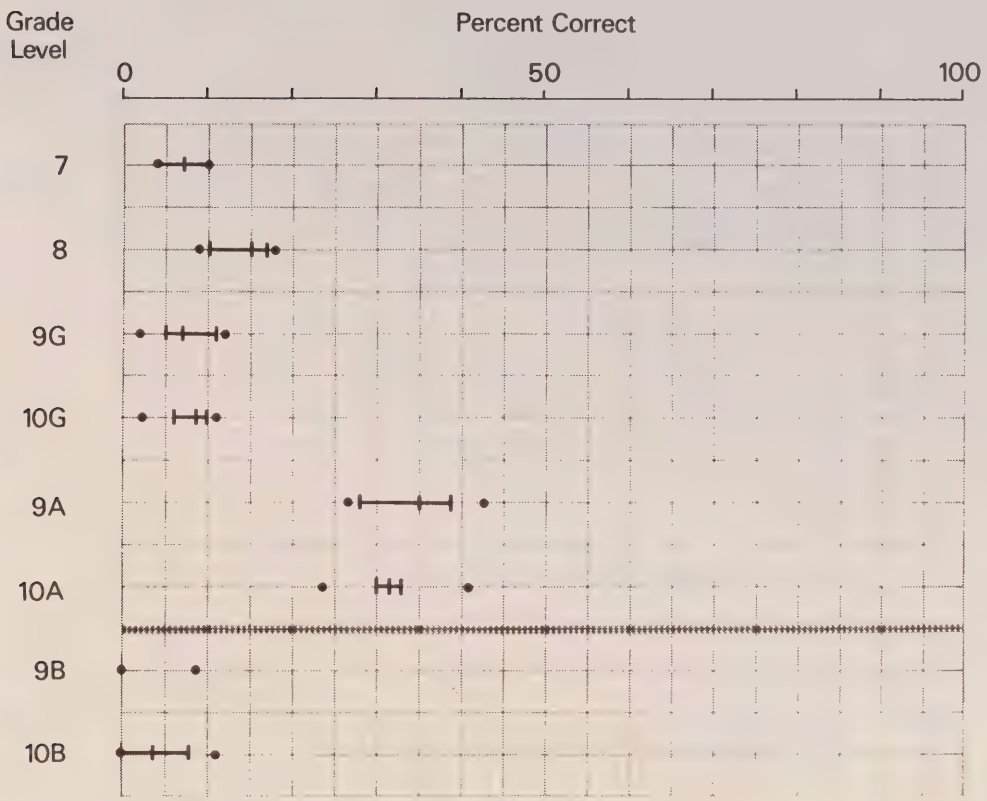
63 =

Display 2.1

FACTORING AND EXPONENTS

Objective: Factoring a Number into Prime Factors (N 005 400; p. 35)

Range of Correct Responses to the Six Instruments, by Grade:



Grade Level	T-Mean %Cor.	T-Mean %Omits
7	7	53
8	15	46
9G	7	53
10G	9	53
9A	35	26
10A	32	32
9B	0	72
10B	4	58

- students throughout the Intermediate Division appear unable to express a number as the product of its prime factors. Almost half of those in the sample chose not to attempt it.
- some of the low performance may have been caused by a misunderstanding of the terminology used in the instrument.
- this topic should be tied more closely to applications such as square root, highest common factor, lowest common multiple.

FACTORING AND EXPONENTS

Objective: Finding Roots of Perfect Squares Using Prime Factors

(N 006 200; p. 38)

Instruments Included in the Field Trial:

(1) The square root of 144 =

(2) The square root of 400 =

(3) The square root of 81 =

(4) The square root of 576 =

(5) The square root of 1225 =

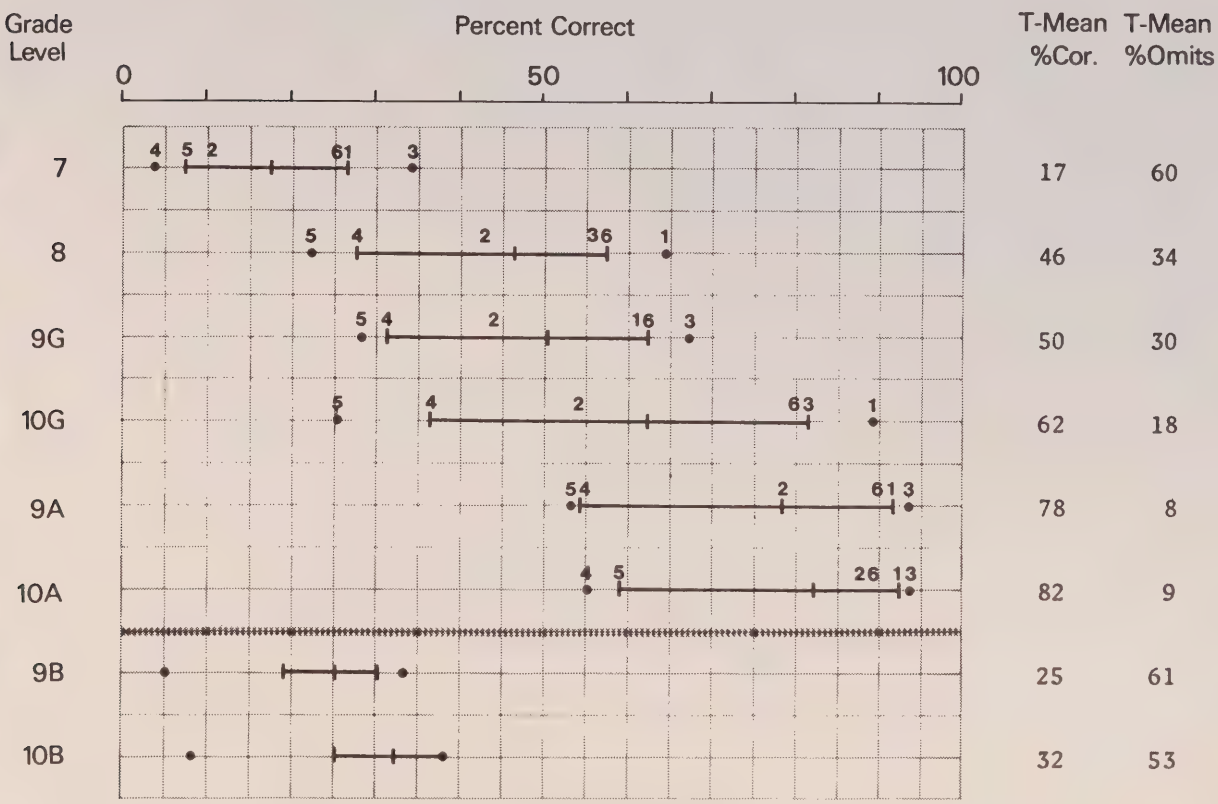
(6) The square root of 64 =

FACTORING AND EXPONENTS

Objective: Finding Roots of Perfect Squares Using Prime Factors

(N 006 200; p. 38)

Range of Correct Responses to the Six Instruments, by Grade:



- the instruments do not reveal whether or not the student used prime factors to find square root.
- results indicate a steady improvement in finding square root throughout the Intermediate Division reaching a high level of performance in Gr. 9 and 10 Advanced classes.
- decreasing number of omits seems to indicate increasing confidence on the part of the students.

Objective: Multiplication and Division of Powers with the Same Base

(N 007 200; p. 41)

Instruments Included in the Field Trial:

(1) Rewrite the following expression as a power

$33^2 \times 33^8 =$

(2) Rewrite the following expression as a power

$\frac{64^8}{64^1} =$

(3) Rewrite the following expression as a power

$86^4 \times 86^8 =$

(4) Rewrite the following expression as a power

$\frac{68^5}{68^3} =$

(5) Rewrite the following expression as a power

$8^5 \times 8^{12} =$

(6) Rewrite the following expression as a power

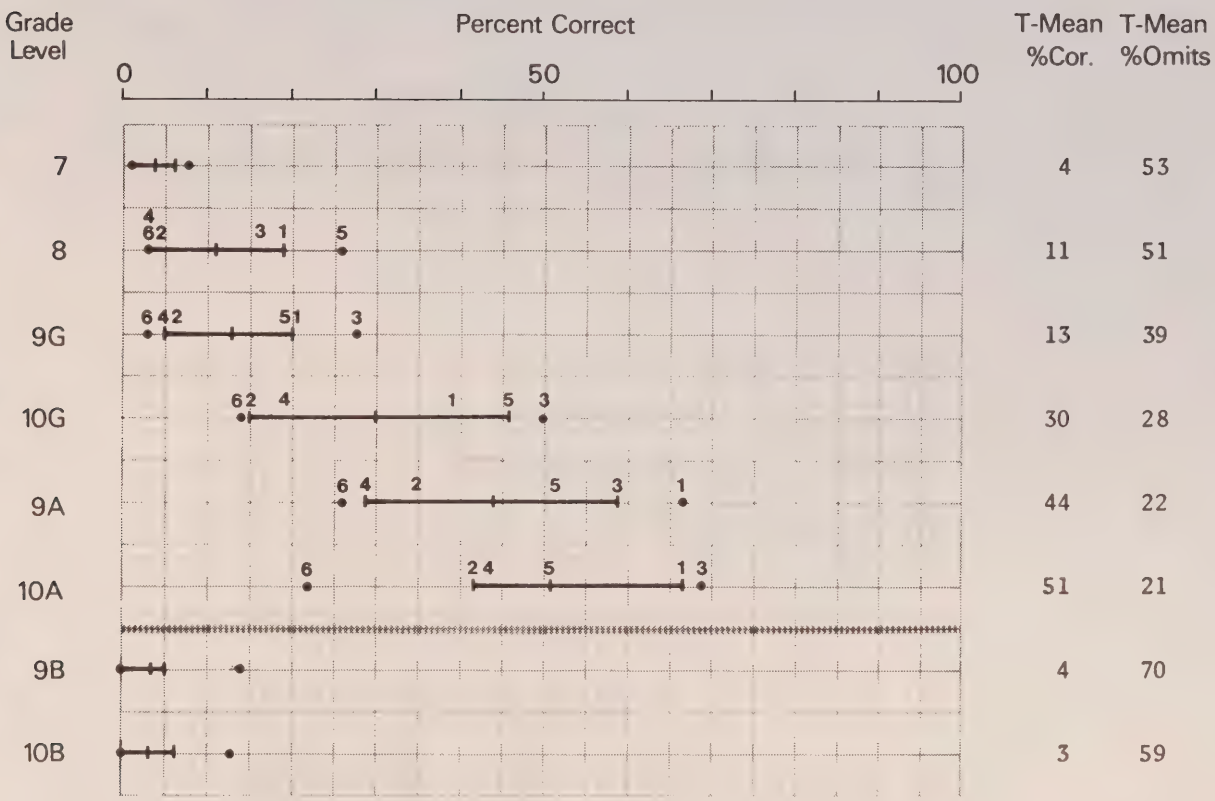
$\frac{4^{15}}{4^5} =$

FACTORING AND EXPONENTS

Objective: Multiplication and Division of Powers with the Same Base

(N 007 200; p. 41)

Range of Correct Responses to the Six Instruments, by Grade:



- the performance of students reflects the relative emphasis of this topic in the guideline. It is core only for Grade 9 Advanced classes and optional for Gr. 7, 8, 9G and 10G.
- there is more extensive work on this topic in Grades 11 and 12.

FACTORING AND EXPONENTS

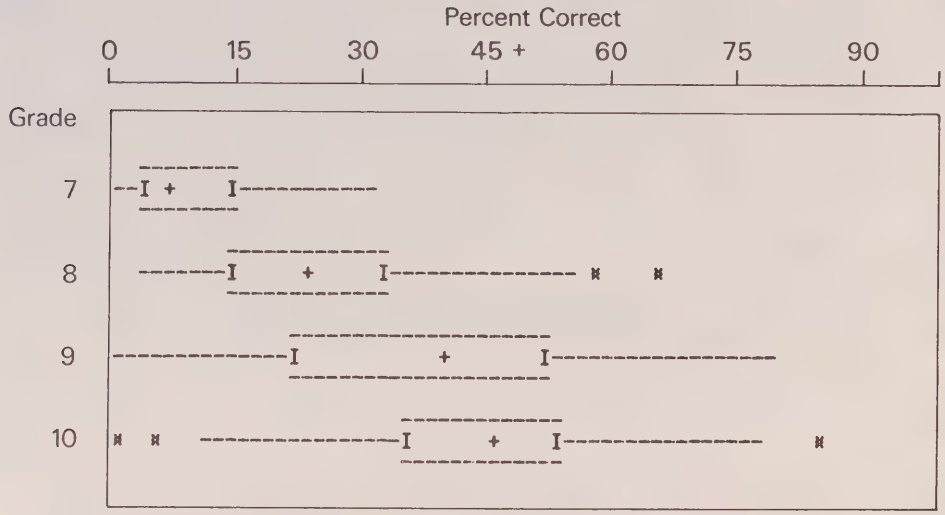
Terminal Objectives in Topic Group

Factoring a Number into Prime Factors (N 005 400; p. 35)

Finding Roots of Perfect Squares Using Prime Factors (N 006 200; p. 38)

Multiplication and Division of Powers with the Same Base (N 007 200; p. 41)

Range of Correct Responses to Topic Group, by Grade:



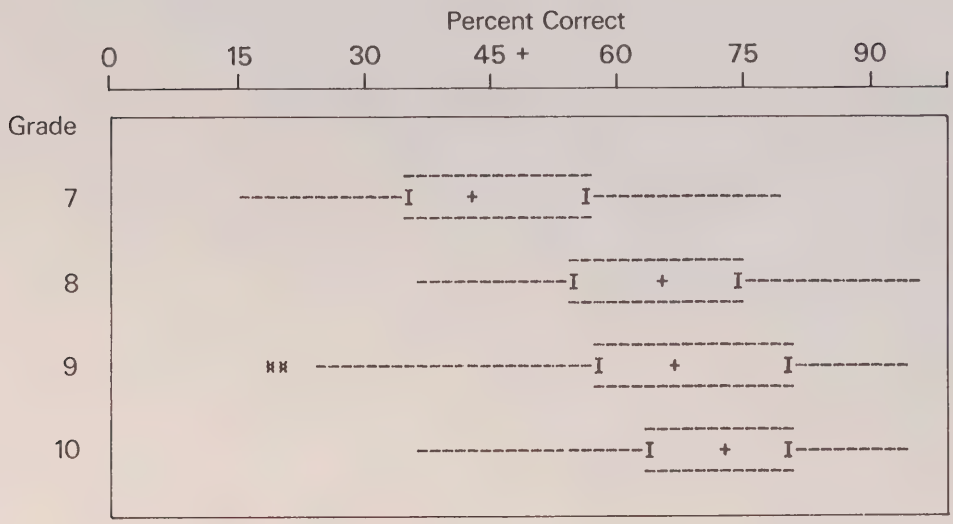
Statistical Summary

Grade Level	No. of Classes	Grade Mean	Grade St. Dev.
7	83	9.2	8.0
8	90	23.9	12.6
9	93	38.0	19.9
10	78	44.2	16.0

FACTORING AND EXPONENTS

Enabling Objectives in Topic Group

Range of Correct Responses to Topic Group, by Grade:



Statistical Summary

Grade Level	No. of Classes	Grade Mean	Grade St. Dev.
7	82	46.7	14.2
8	88	66.0	13.6
9	89	68.0	17.7
10	74	72.6	12.8

The results displayed in this graph are based on instruments developed from the following objectives in the published pool:

N 005 100: Determining if one number is a factor of another (p. 30)
N 005 200: Listing all the factors of a given number (p. 31)
N 006 100: Finding the roots of perfect squares by inspection (p. 37)
N 007 100: Exponents (p. 40)

Objective: Order of Operations (N 008 200; p. 44)

Instruments Included in the Field Trial:

(1) $2 \times (36 - (6 + 3)) =$

(2) $(5 + 6^3) + 3 =$

(3) $\frac{(33 + 3)}{3} =$

(4) $6 + (30 - (2 + 7)) \times 3 =$

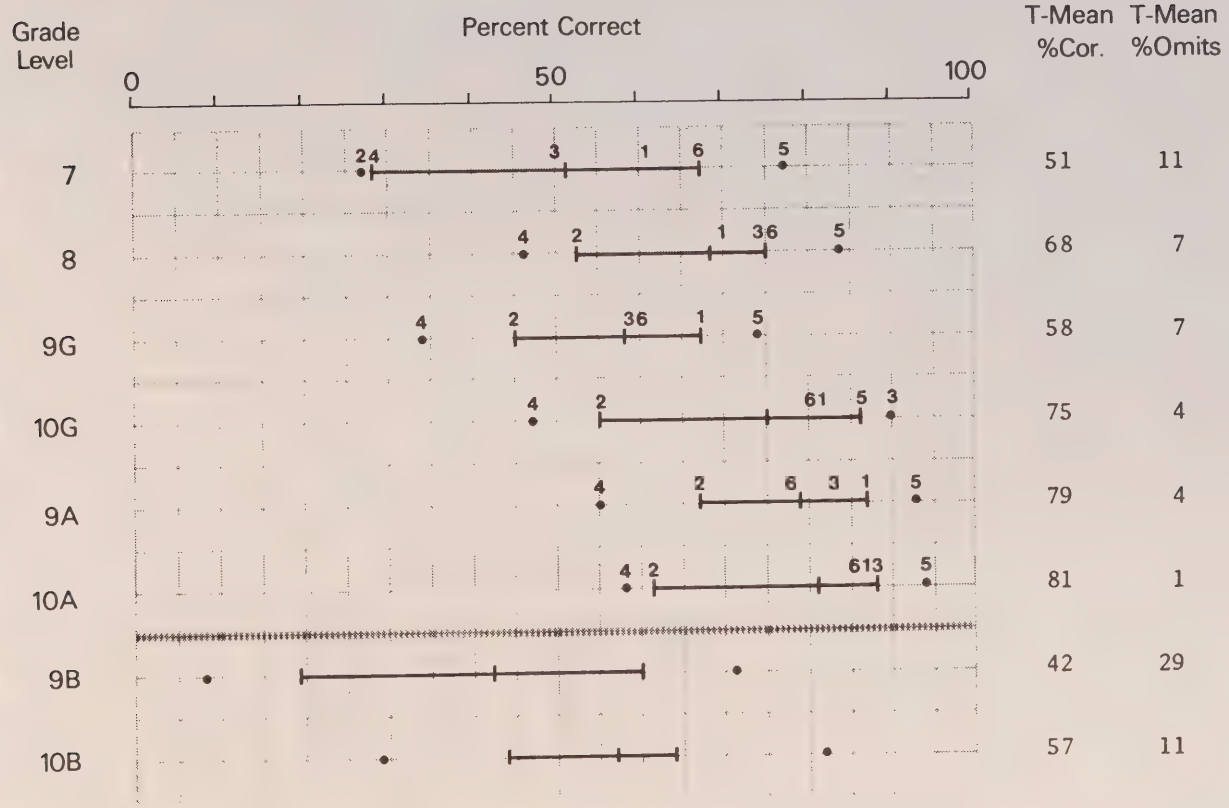
(5) $30 - (4 + 5) =$

(6) $(30 - 5) \times (2 + 6) =$

ORDER OF OPERATIONS

Objective: Order of Operations (N 008 200; p. 44)

Range of Correct Responses to the Six Instruments, by Grade:



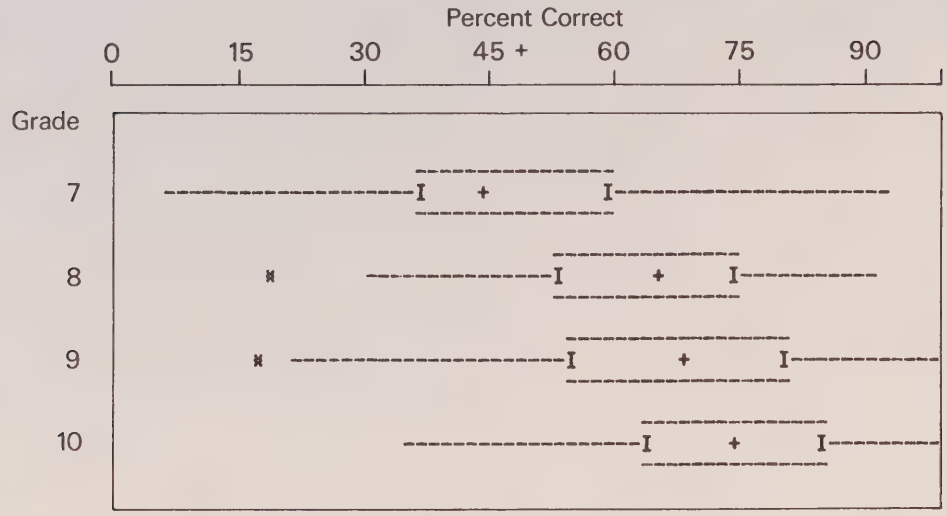
- these 'skill testing questions' are more important in our province than prime factors.
- there is a continued improvement in performance across the Intermediate Division resulting in a high level of achievement by the end of Grade 10.

ORDER OF OPERATIONS

Terminal Objective in Topic Group

Order of Operations (N 008 200; p. 44)

Range of Correct Responses to Topic Group, by Grade:



Statistical Summary

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8	91	63.8	13.8
9	91	67.8	18.7
10	75	75.3	14.7

OPERATIONS WITH DECIMALS

Objective: Adding and Subtracting Numbers From 0.001 to 900

(N 009 100; p. 46)

Instruments Included in the Field Trial:

(1) 0.041 + 63.1 + 6 =

(2) 879 - 2.84 =

(3) 0.6 + 0.08 + 0.24 =

(4) 5 - 0.3 =

(5) 0.345 + 6 + 150 =

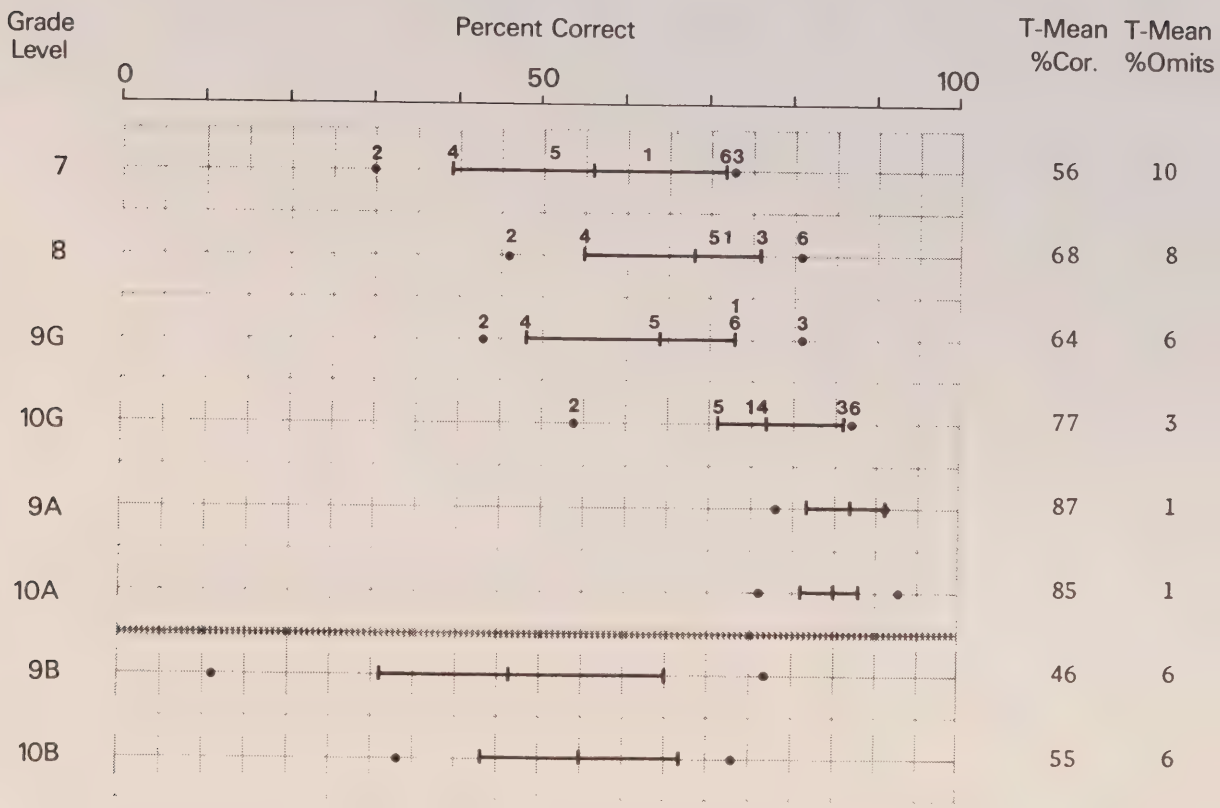
(6) 59.2 - 0.7 =

OPERATIONS WITH DECIMALS

Objective: Adding and Subtracting Numbers From 0.001 to 900

(N 009 100; p. 46)

Range of Correct Responses to the Six Instruments, by Grade:



- there is good progress across the grade levels although the performance of Gr. 9 and 10 Basic classes is disappointing.
- the format of instruments may have contributed to the fairly broad range of achievement.

OPERATIONS WITH DECIMALS

Objective: Multiplying by Powers of 10 (N 010 200; p. 50)

Instruments Included in the Field Trial:

(1) $0.04209 \times 100 =$

(2) $0.02714 \times 10 =$

(3) $0.7218 \times 100 =$

(4) $0.01212 \times 10 =$

(5) $35.06 \times 1000 =$

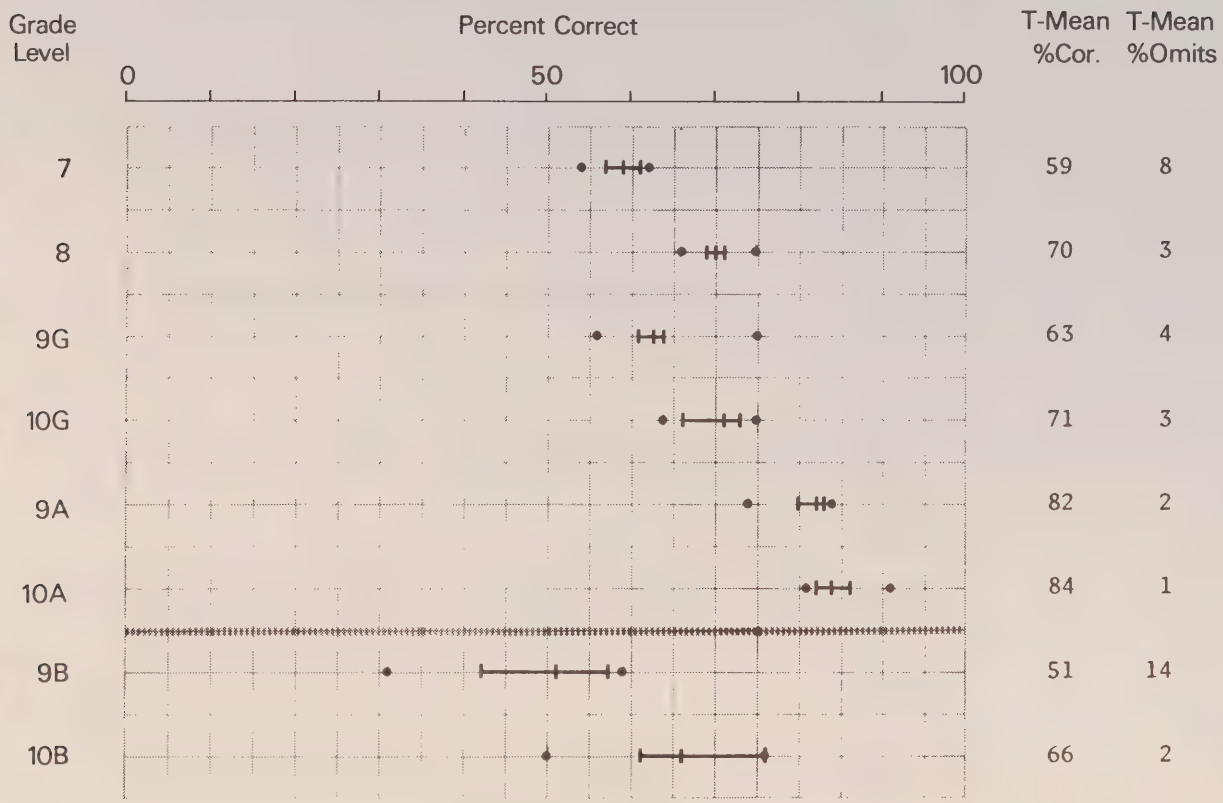
(6) $0.0912 \times 10 =$

Display 4.2

OPERATIONS WITH DECIMALS

Objective: Multiplying by Powers of 10 (N 010 200; p. 50)

Range of Correct Responses to the Six Instruments, by Grade:



- the good performance on these instruments apparently reflects a narrow range of specific objectives.
- the objective is specified as core for Grade 7.

Objective: Dividing with Decimal Divisors (N 011 200; p. 53)

Instruments Included in the Field Trial:

(1) Rewrite the following fraction as a decimal (rounded to two decimal places if necessary)

$\frac{18.7}{2.3} =$

(2) Rewrite the following fraction as a decimal (rounded to two decimal places if necessary)

$\frac{2.97}{0.54} =$

(3) Rewrite the following fraction as a decimal (rounded to two decimal places if necessary)

$\frac{3}{0.8} =$

(4) Rewrite the following fraction as a decimal (rounded to two decimal places if necessary)

$\frac{28.6}{9.9} =$

(5) Rewrite the following fraction as a decimal (rounded to two decimal places if necessary)

$\frac{3.72}{0.71} =$

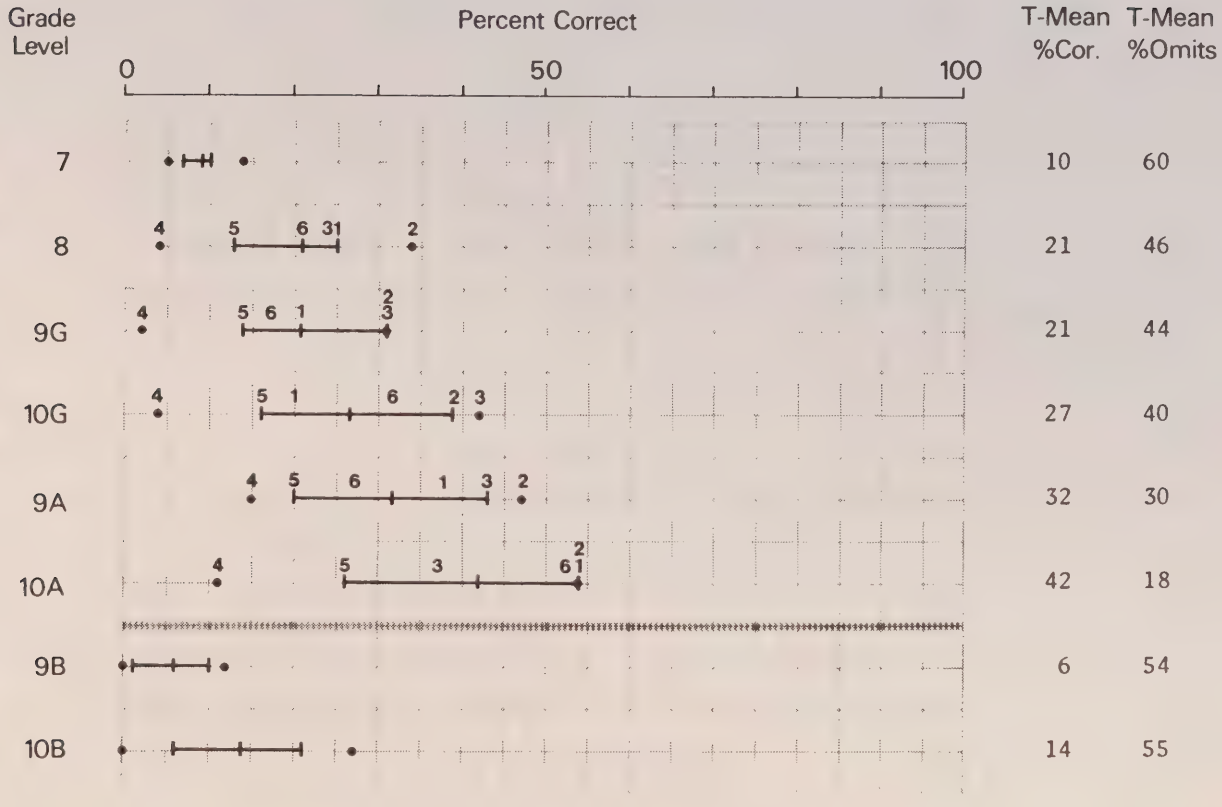
(6) Rewrite the following fraction as a decimal (rounded to two decimal places if necessary)

$\frac{1.5}{0.6} =$

OPERATIONS WITH DECIMALS

Objective: Dividing with Decimal Divisors (N 011 200; p. 53)

Range of Correct Responses to the Six Instruments, by Grade:



- achievement is poor on these instruments.
- the format of the instruments is not commonly used in classrooms. Performance may have been better had the instruments appeared as 9.9/28.6.
- rounding skills are being tested as well as division.
- these instruments should be rewritten so that they test only the objective specified.

OPERATIONS WITH DECIMALS

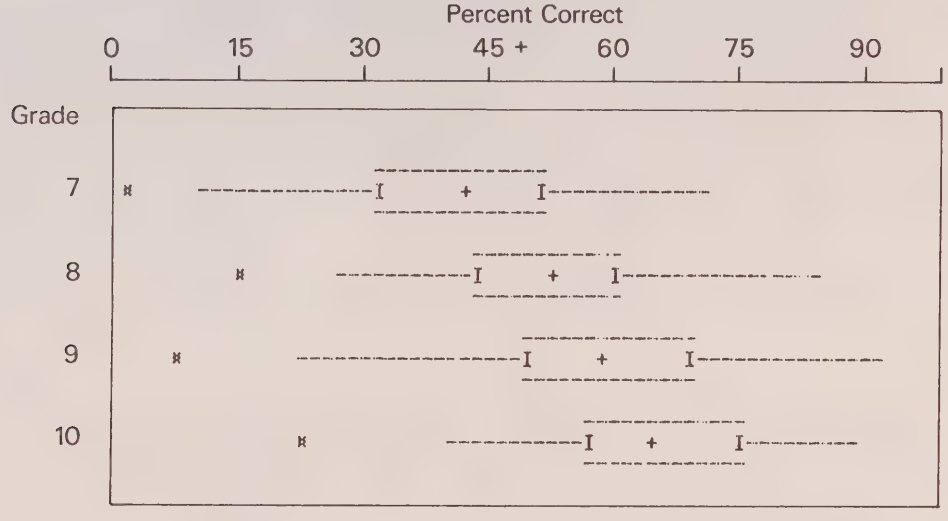
Terminal Objectives in Topic Group

Adding and Subtracting Numbers From 0.001 to 900 (N 009 100; p. 46)

Multiplying by Powers of 10 (N 010 200; p. 50)

Dividing with Decimal Divisors (N 011 200; p. 53)

Range of Correct Responses to Topic Group, by Grade:



Statistical Summary

Grade Level	No. of Classes	Grade Mean	Grade St. Dev.
7	90	41.0	14.4
8	95	53.2	11.8
9	99	57.9	15.6
10	89	65.2	12.6

Objective: Multiplying Two Mixed Numbers (N 012 600; p. 62)

Instruments Included in the Field Trial:

(1) Simplify the following expression

$$1\frac{3}{4} \times 2\frac{1}{2} =$$

(2) Simplify the following expression

$$2\frac{3}{4} \times 1\frac{2}{3} =$$

(3) Simplify the following expression

$$3\frac{1}{4} \times 2\frac{2}{3} =$$

(4) Simplify the following expression

$$1\frac{1}{5} \times 2\frac{3}{4} =$$

(5) Simplify the following expression

$$1\frac{4}{5} \times 3\frac{1}{4} =$$

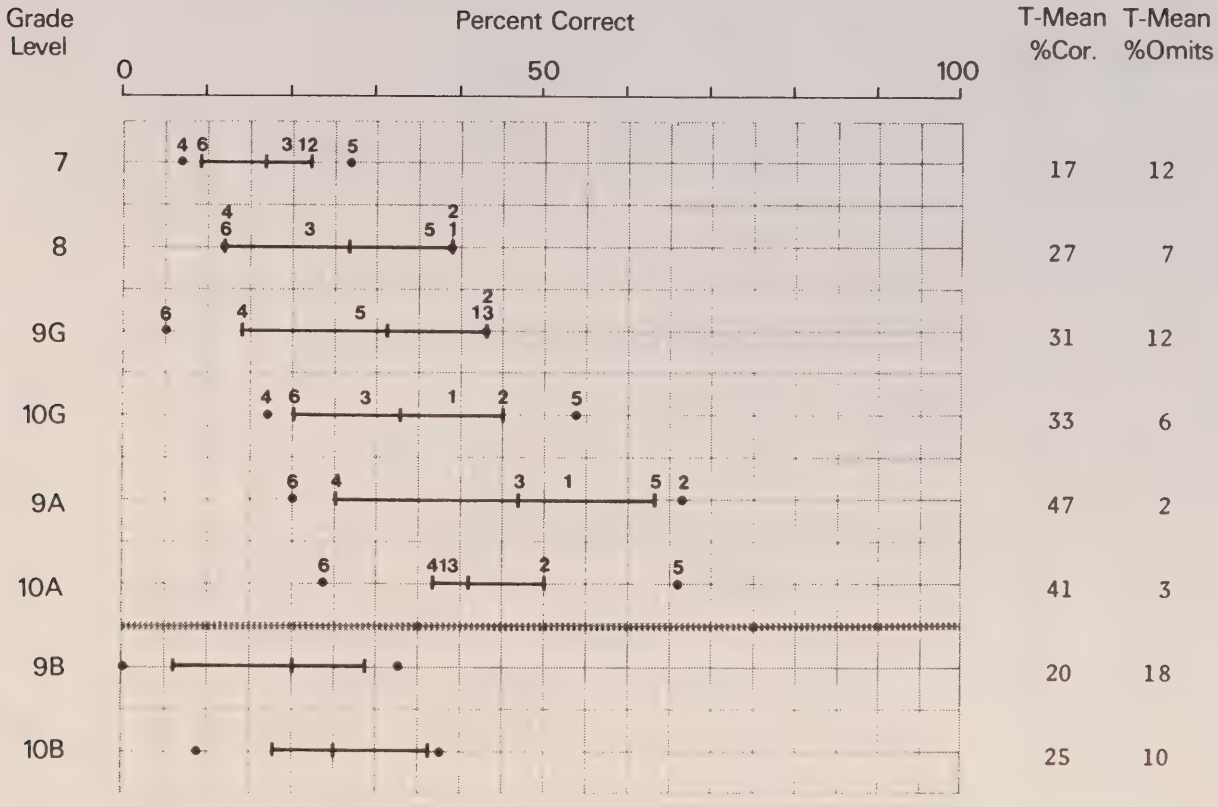
(6) Simplify the following expression

$$3\frac{4}{5} \times 3\frac{2}{3} =$$

OPERATIONS WITH FRACTIONS

Objective: Multiplying Two Mixed Numbers (N 012 600; p. 62)

Range of Correct Responses to the Six Instruments, by Grade:



- the performance of students is very disappointing, but probably reflects the low priority placed on this topic by teachers.

Objective: Dividing a Mixed Number by a Natural Number/by a Fraction

(N 013 100/200; p. 64/65)

Instruments Included in the Field Trial:

(1) Simplify the following expression

$$\frac{1\frac{1}{2}}{\frac{1}{2}} =$$

(2) Simplify the following expression

$$\frac{\frac{1}{2}}{\frac{2}{3}} =$$

(3) Simplify the following expression

$$\frac{1\frac{1}{2}}{2\frac{3}{5}} =$$

(4) Simplify the following expression

$$\frac{2\frac{1}{2}}{2} =$$

(5) Simplify the following expression

$$\frac{1\frac{3}{5}}{\frac{2}{3}} =$$

(6) Simplify the following expression

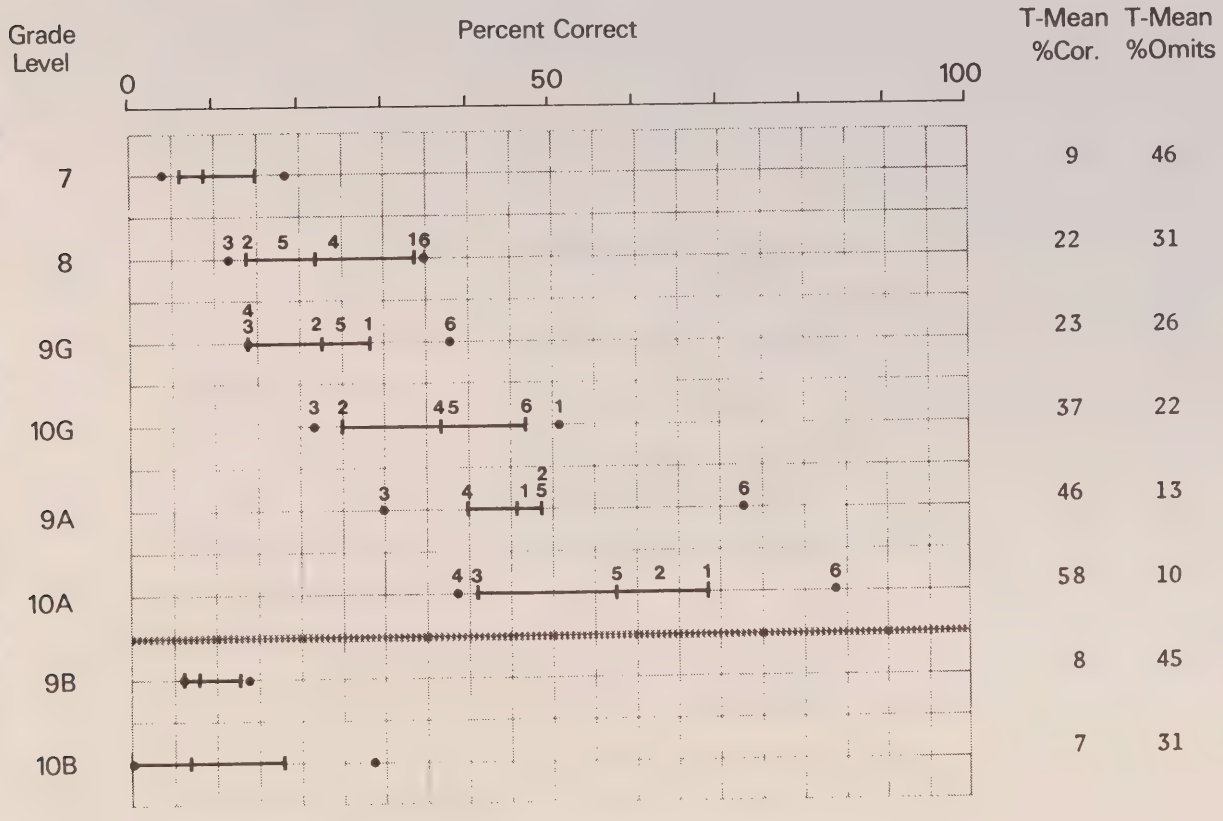
$$\frac{\frac{3}{4}}{\frac{1}{4}} =$$

OPERATIONS WITH FRACTIONS

Objective: Dividing a Mixed Number by a Natural Number/by a Fraction

(N 013 100/200; p. 64/65)

Range of Correct Responses to the Six Instruments, by Grade:



- the performance of students shows good progress for division of proper fractions but not for division of mixed numbers.
- division of mixed numbers is not a skill requiring emphasis.
- the format of the model should be revised so that mixed numbers do not occur and also so that proper fractions appear in the form $\frac{a}{b} \div \frac{c}{d}$.

Objective: Adding and Subtracting Mixed Numbers (N 014 300; p. 70)

Instruments Included in the Field Trial:

(1) Simplify the following expression

$2\frac{3}{5} - 1\frac{1}{2} =$

(2) Simplify the following expression

$1\frac{1}{2} + 4\frac{1}{3} =$

(3) Simplify the following expression

$2\frac{1}{4} - 1\frac{3}{5} =$

(4) Simplify the following expression

$1\frac{1}{5} + 4\frac{1}{4} =$

(5) Simply the following expression

$3\frac{2}{5} - 1\frac{2}{3} =$

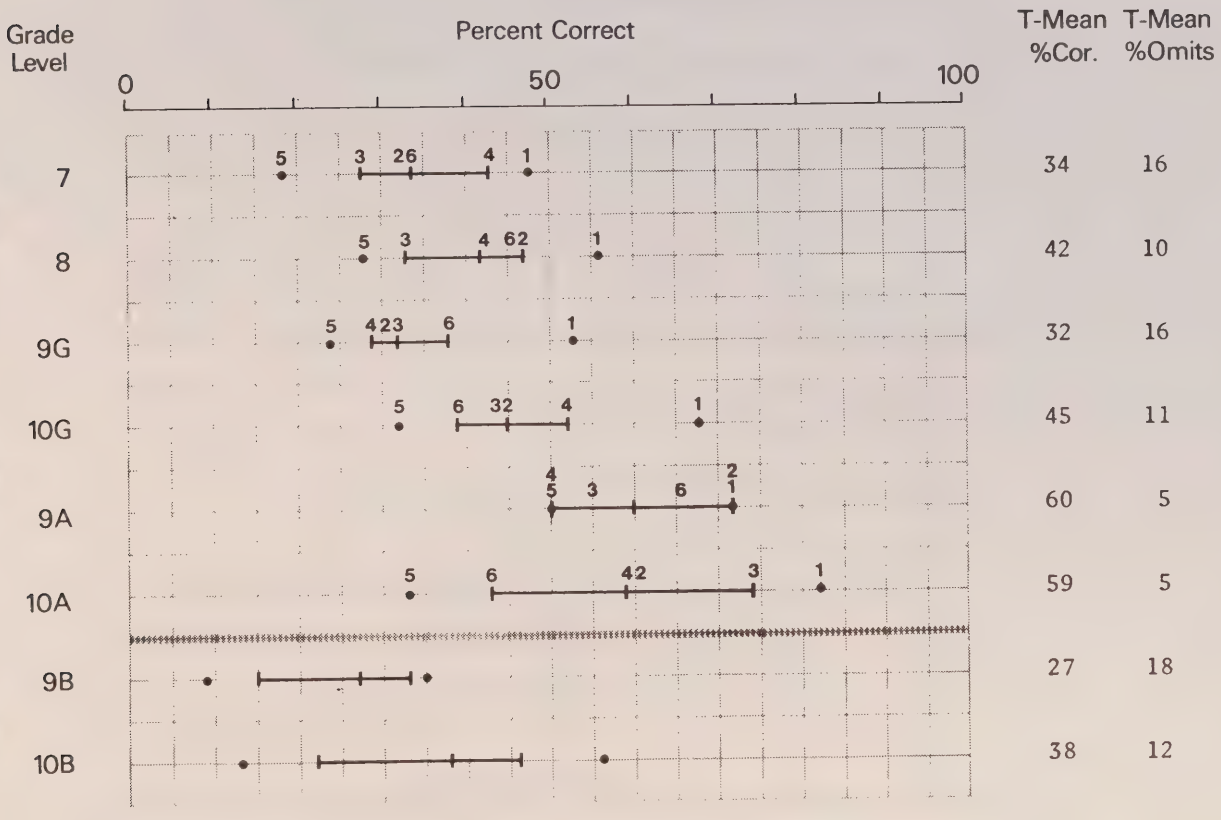
(6) Simplify the following expression

$2\frac{2}{5} + 2\frac{1}{2} =$

OPERATIONS WITH FRACTIONS

Objective: Adding and Subtracting Mixed Numbers (N 014 300; p. 70)

Range of Correct Responses to the Six Instruments, by Grade:



- the progress of students across the Intermediate Division is reasonable for this objective.
- objectives should be reviewed so that appropriate sequencing of the skills involved is ensured.
- users are advised to include instruments assessing performance on the enabling objective, Adding Proper Fractions.

OPERATIONS WITH FRACTIONS

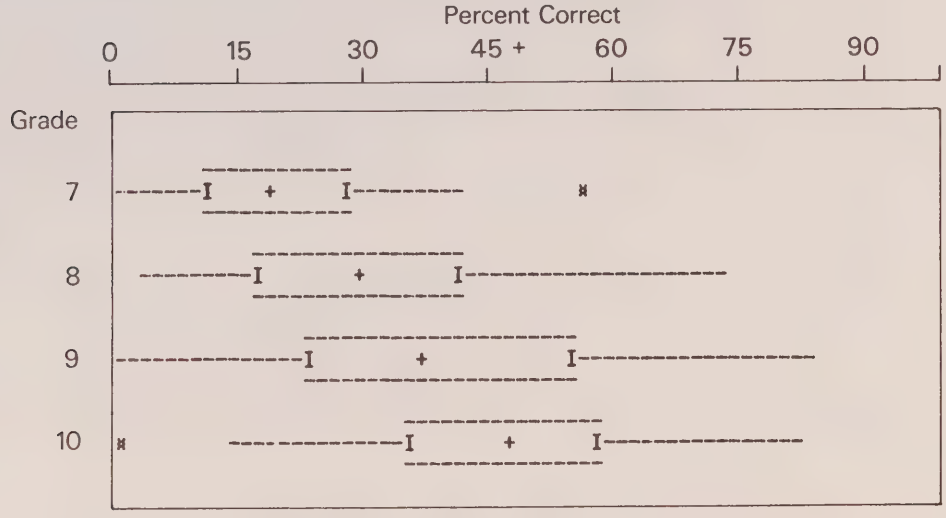
Terminal Objectives in Topic Group

Multiplying Two Mixed Numbers (N 012 600; p. 62)

Multiplying a Mixed Number by a Natural Number/by a Fraction (N 013 100/200; p. 64/65)

Adding and Subtracting Mixed Numbers (N 014 300; p. 70)

Range of Correct Responses to Topic Group, by Grade:



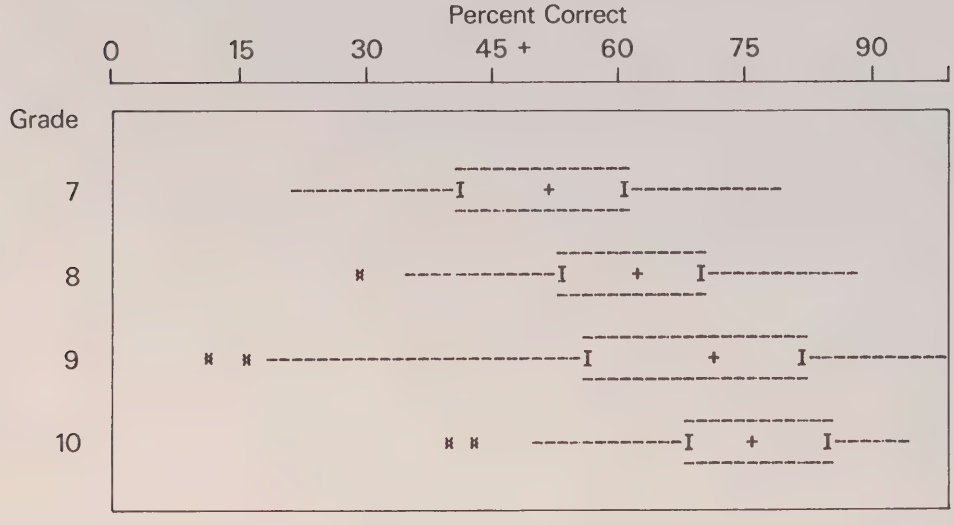
Statistical Summary

Grade Level	No. of Classes	Grade Mean	Grade St. Dev.
7	95	20.1	11.1
8	97	30.3	15.2
9	109	38.2	19.2
10	98	46.4	15.6

OPERATIONS WITH FRACTIONS

Enabling Objectives in Topic Group

Range of Correct Responses to Topic Group, by Grade:



Statistical Summary

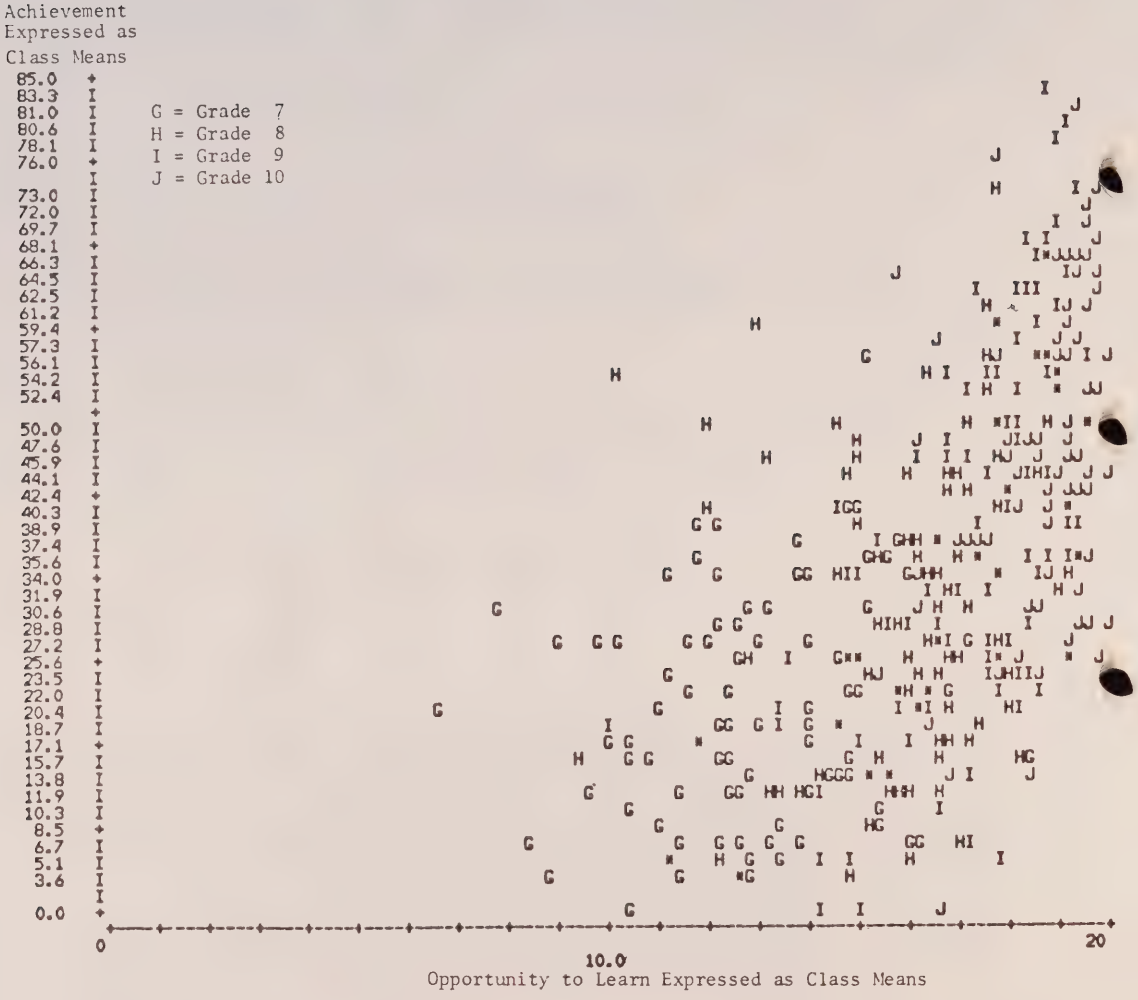
Grade Level	No. of Classes	Grade Mean	Grade St. Dev.
7	96	50.9	14.1
8	98	61.0	12.0
9	116	67.3	18.6
10	98	75.2	11.8

The results presented in this graph are based on instruments developed from the following objectives in the published pool:

N 012 100: Multiplying unit fractions by natural numbers (p. 56)
N 012 200: Reducing fractions to lowest terms (p. 58)
N 012 300: Multiplying two fractions (p. 59)
N 012 400: Changing mixed number to fraction form (p. 60)
N 012 500: Multiplying mixed numbers by fractions (p. 61)
N 014 100: Adding and subtracting fractions with the same denominators (p. 68)
N 014 200: Adding and subtracting fractions with unlike denominators (p. 69)

OPERATONS WITH FRACTIONS

Relationship Between Achievement and Opportunity to Learn:



RELATING FRACTIONS AND DECIMALS

Objective: Conversion of Decimal Fractions to Familiar Common Fractions

(N 015 100; p. 72)

Instruments Included in the Field Trial:

(1) Convert the following to a mixed number or a common fraction. Reduce all fractions to lowest terms.

1.5 =

(2) Convert the following to a mixed number or a common fraction. Reduce all fractions to lowest terms.

2.6 =

(3) Convert the following to a mixed number or a common fraction. Reduce all fractions to lowest terms.

0.9 =

(4) Convert the following to a mixed number or a common fraction. Reduce all fractions to lowest terms.

2.3 =

(5) Convert the following to a mixed number or a common fraction. Reduce all fractions to lowest terms.

0.375 =

(6) Convert the following to a mixed number or a common fraction. Reduce all fractions to lowest terms.

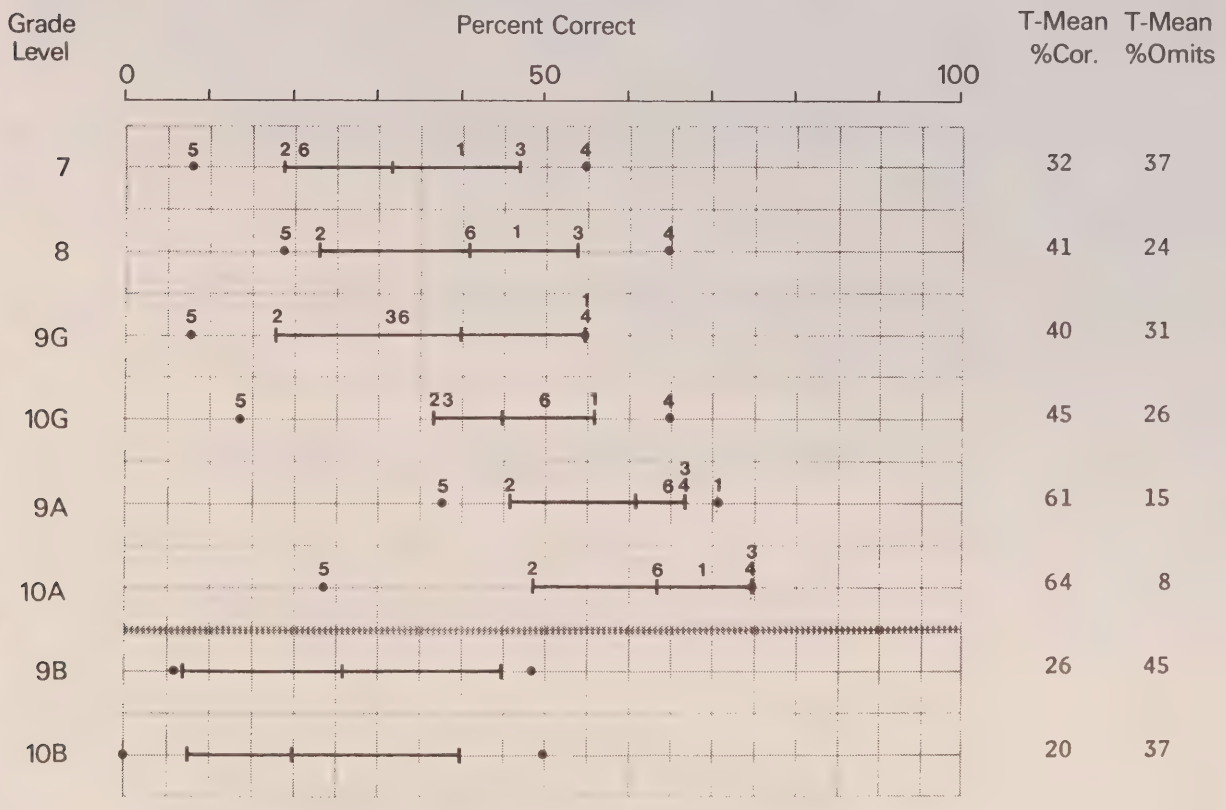
2.25 =

RELATING FRACTIONS AND DECIMALS

Objective: Conversion of Decimal Fractions to Familiar Common Fractions

(N 015 100; p. 72)

Range of Correct Responses to the Six Instruments, by Grade:



- there is not much improvement in performance across the grade levels.
- this topic is not emphasized in the curriculum.
- the model generates instruments that are clearly at different levels of difficulty. Some require reduction to lowest terms.
- objectives should be reviewed so that appropriate sequencing of the skills involved is ensured.

RELATING FRACTIONS AND DECIMALS

Objective: Converting Common Fractions to Decimal Fractions

(N 016 100; p. 75)

Instruments Included in the Field Trial:

(1) Convert the following fraction to decimal form. Round your answer to three decimal places.

$\frac{28}{5}$

(2) Convert the following fraction to decimal form. Round your answer to three decimal places.

$\frac{2}{27}$

(3) Convert the following fraction to decimal form. Round your answer to three decimal places.

$\frac{271}{33}$

(4) Convert the following fraction to decimal form. Round your answer to three decimal places.

$\frac{17}{23}$

(5) Convert the following fraction to decimal form. Round your answer to three decimal places.

$\frac{147}{44}$

(6) Convert the following fraction to decimal form. Round your answer to three decimal places.

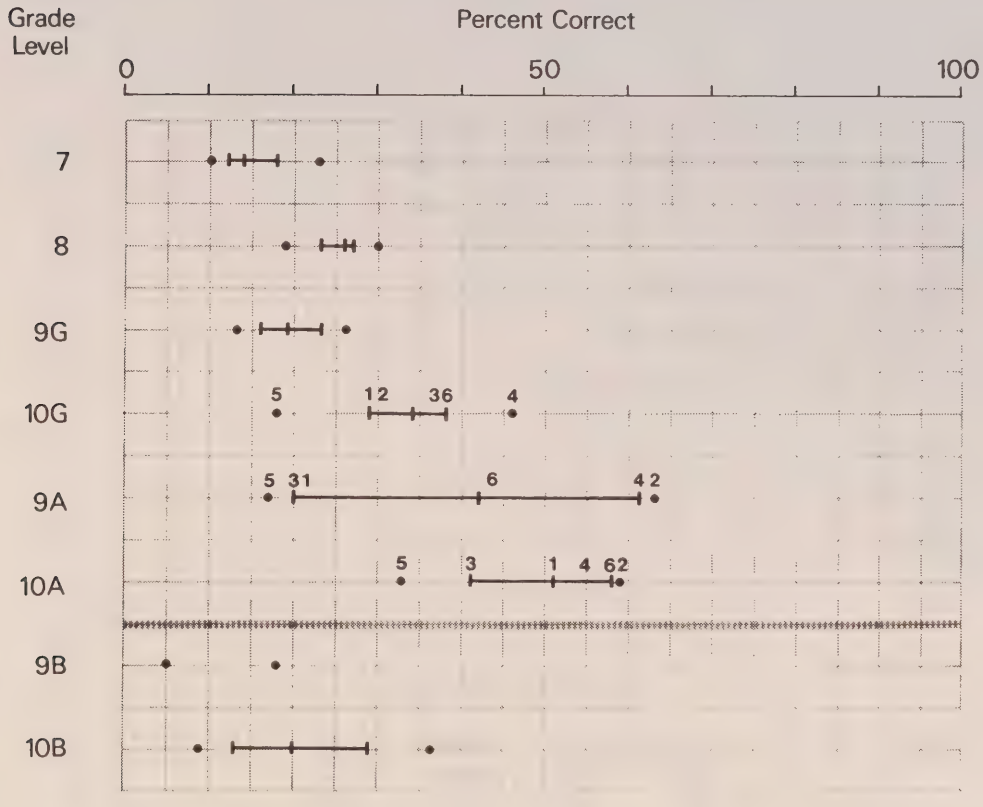
$\frac{2}{37}$

RELATING FRACTIONS AND DECIMALS

Objective: Converting Common Fractions to Decimal Fractions

(N 016 100; p. 75)

Range of Correct Responses to the Six Instruments, by Grade:



- there is little improvement on the rather low level of performance throughout the Intermediate Division
- this skill will be more important with the use of calculators.
- the instruments do not always test the skill at a suitable level of difficulty for the students.
- the model should be revised so that it will generate items that test the skill at a less complex level.
- objectives should be reviewed so that appropriate sequencing of the skills involved is ensured.

RELATING FRACTIONS AND DECIMALS

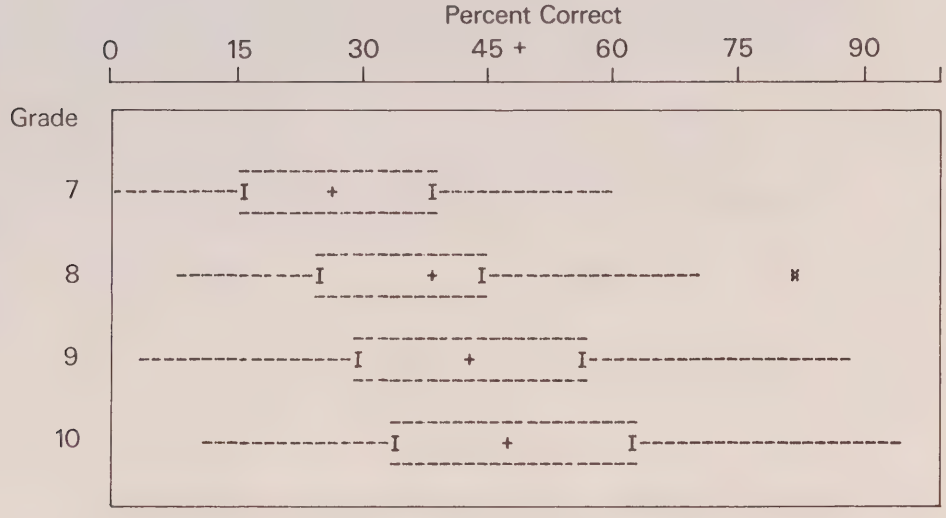
Terminal Objectives in Topic Group

Converions of Decimal Fractions to Familiar Common Fractions

(N 015 100; p. 72)

Converting Common Fractions to Decimal Fractions (N 016 100; p. 75)

Range of Correct Responses to Topic Group, by Grade:

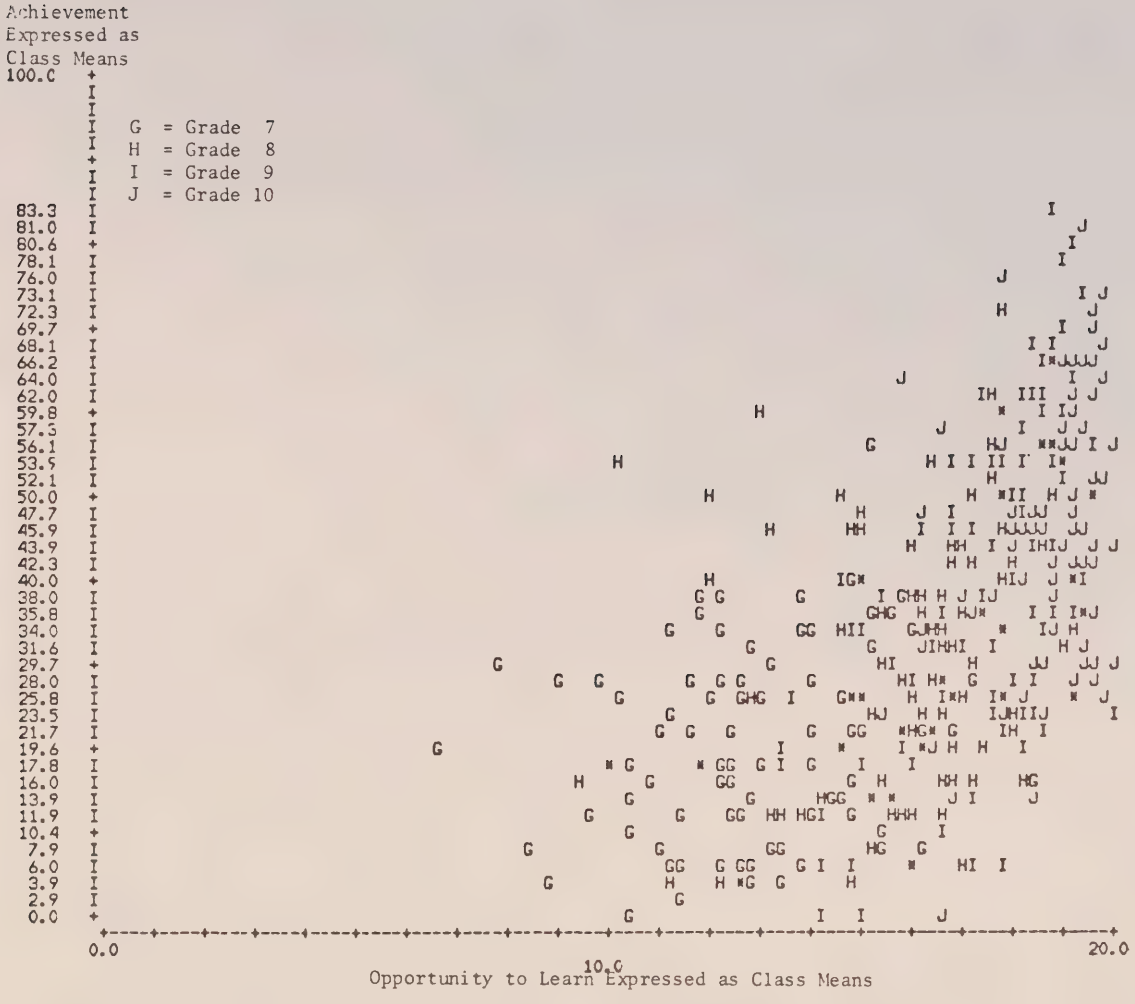


Statistical Summary

Grade Level	No. of Classes	Grade Mean	Grade St. Dev.
7	74	26.4	14.0
8	85	36.2	15.1
9	80	42.1	18.6
10	66	48.7	17.5

RELATING FRACTIONS AND DECIMALS

Relationship Between Achievement and Opportunity to Learn:



TWO TERM RATIOS

Objective: Solving for an Unknown in Equivalent Ratios

(N 017 400; p. 84)

Instruments Included in the Field Trial:

(1) $13:1 = x:10, \quad x =$

(2) $\frac{x}{10} = \frac{4}{1}, \quad x =$

(3) $2:6 = 5:x, \quad x =$

(4) $4:14 = 8:x, \quad x =$

(5) $x:7 = 11:1, \quad x =$

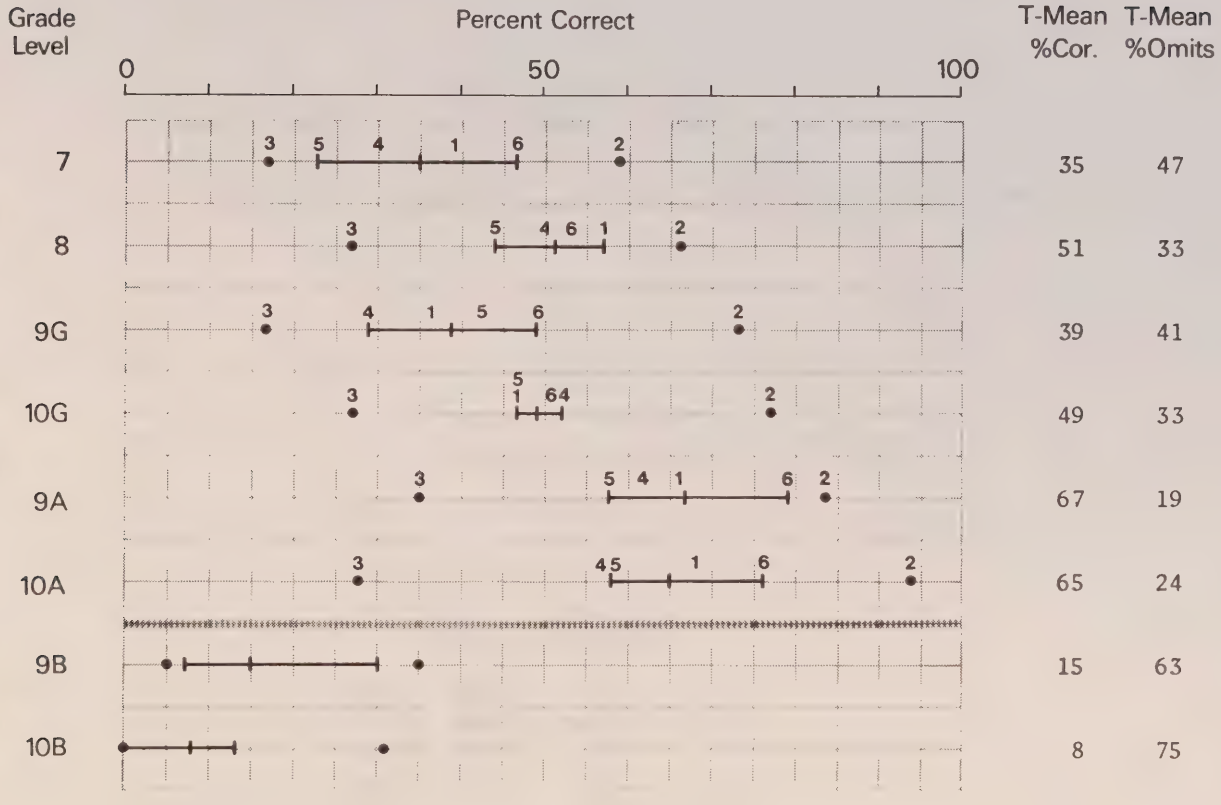
(6) $\frac{8}{x} = \frac{1}{9}, \quad x =$

TWO TERM RATIOS

Objective: Solving for an Unknown in Equivalent Ratios

(N 017 400; p. 84)

Range of Correct Responses to the Six Instruments, by Grade:



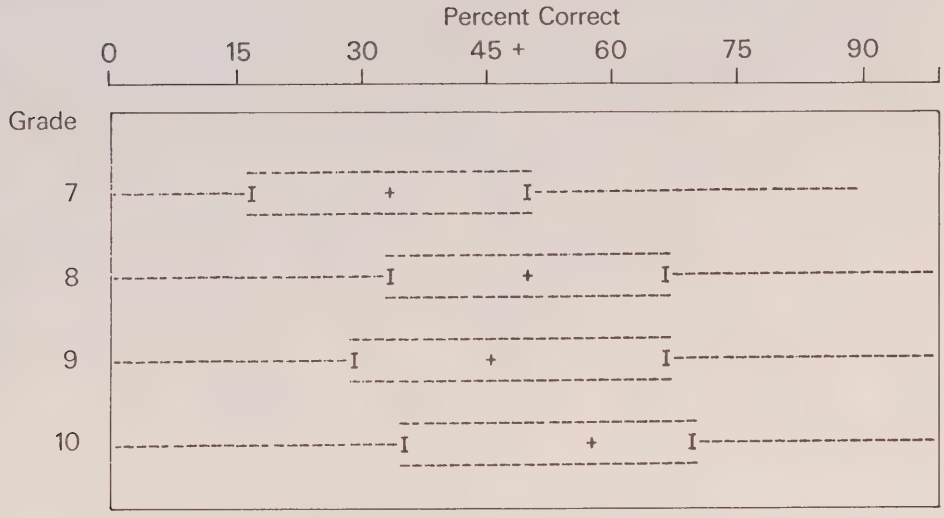
- there appears to be good progress from grade to grade on the simpler instruments.
- the instruments vary widely in difficulty.
- this topic is difficult for students who are just beginning to develop formal abstract thinking.
- the format of the instruments may well have caused some difficulty for pupils.
- the teacher questionnaire indicates that the topic is taught at all grade levels with most teachers using a cross product approach.
- teachers should emphasize a variety of strategies in teaching this topic.
- in-service work on this topic may be required.

TWO TERM RATIOS

Terminal Objective in Topic Group

Solving for an Unknown in Equivalent Ratios (N 017 400; p. 84)

Range of Correct Responses to Topic Group, by Grade:



Statistical Summary

Grade Level	No. of Classes	Grade Mean	Grade St. Dev.
7	97	35.4	24.1
8	98	49.0	23.3
9	119	47.1	27.2
10	107	52.3	24.7

PERCENTAGES

Objective: Converting Percents, Ratios and Decimals (N 018 400; p. 91)

Instruments Included in the Field Trial:

(1) 104% expressed as a decimal fraction =

(2) 36% expressed as a ratio =

(3) $\frac{1}{2}$ expressed as a percent to the nearest tenth =

(4) 148.7% expressed as a ratio =

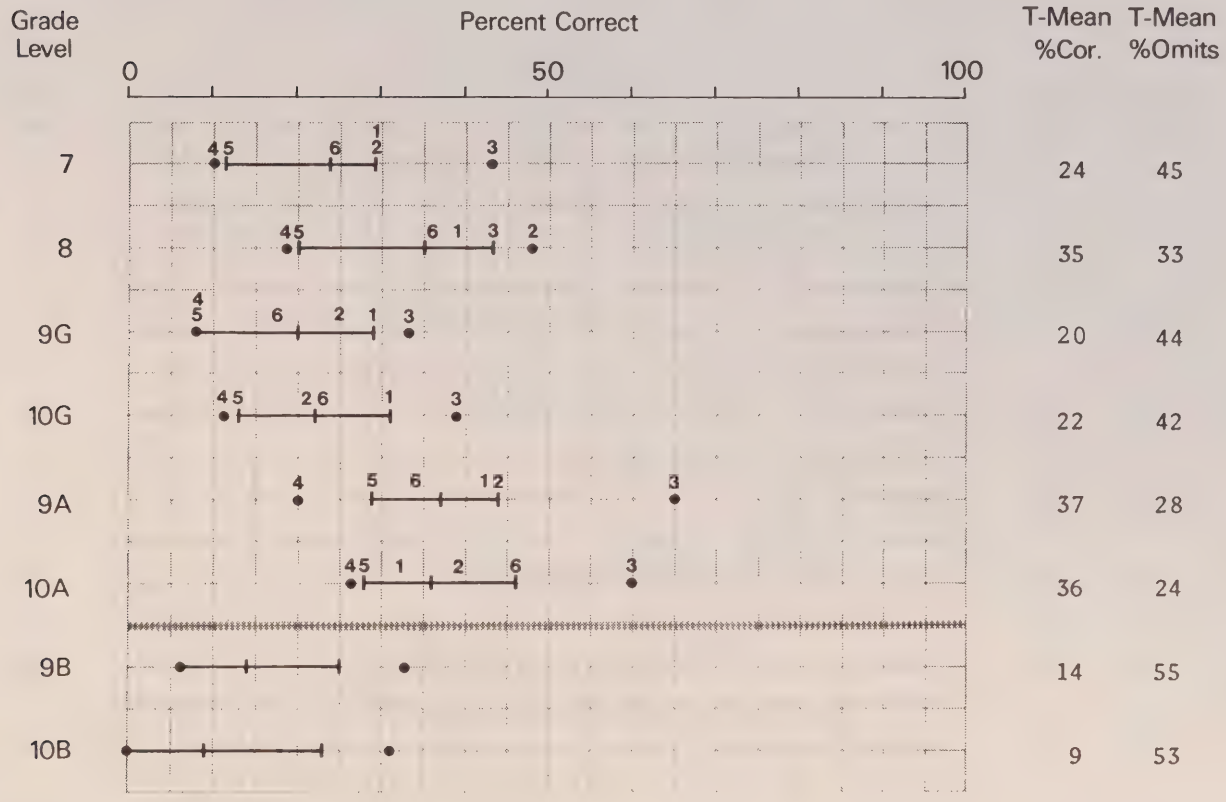
(5) 119.3% expressed as a ratio =

(6) 195% expressed as a ratio =

PERCENTAGES

Objective: Converting Percents, Ratios and Decimals (N 018 400; p. 91)

Range of Correct Responses to the Six Instruments, by Grade:



- achievement is poor and there is relatively little progress throughout the Intermediate Division.
- instruments reflect the composite nature of the title.
- although the pool does contain objectives and appropriate models for generating instruments for each of the three types of conversion, only the objective expressed above was tested.
- Intermediate Division Guidelines need to be much more specific with respect to the objective of this topic.

PERCENTAGES

Objective: Solving a Variety of Percent Problems (N 019 400; p. 97)

Instruments Included in the Field Trial:

(1) 53 is what percent of 91?

(2) 37% of 100 =

(3) 75 out of what number is 105%?

(4) What percent is 18 out of 89?

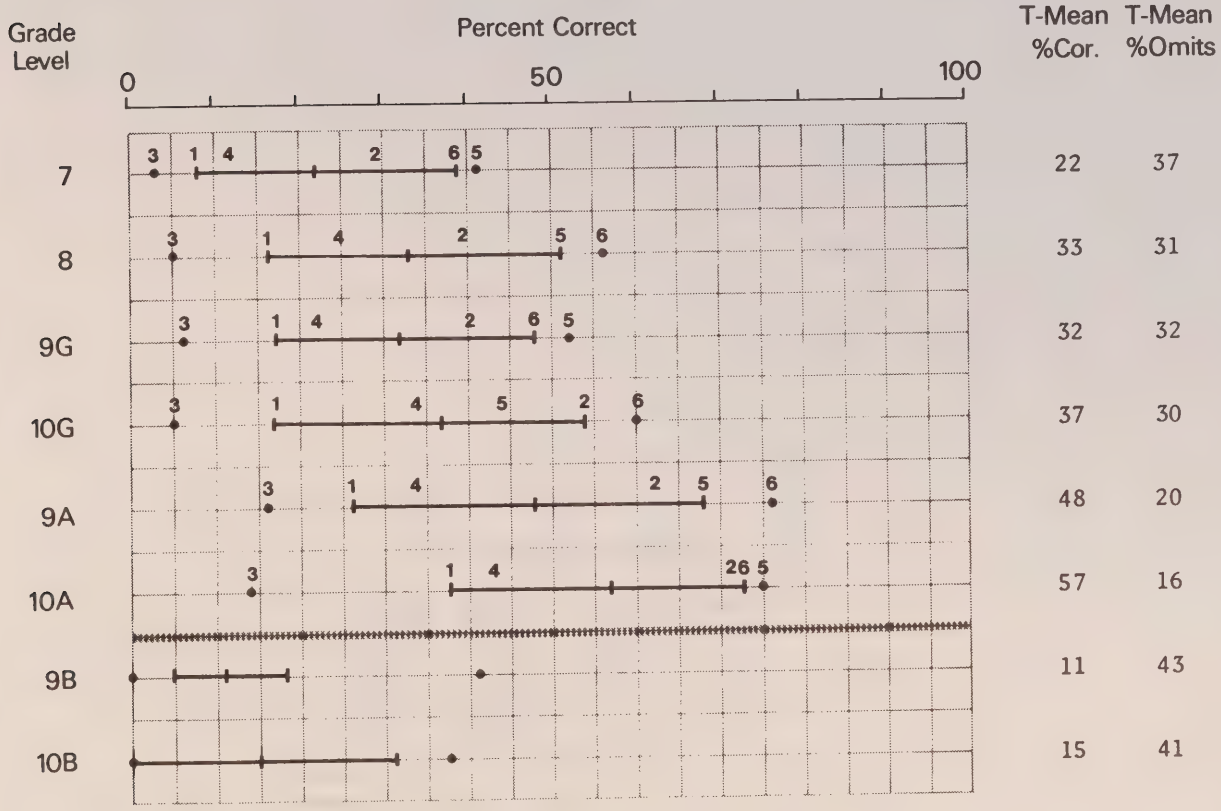
(5) 50% of what number is 12?

(6) 100% of what number is 75?

PERCENTAGES

Objective: Solving a Variety of Percent Problems (N 019 400; p. 97)

Range of Correct Responses to the Six Instruments, by Grade:



- there is a wide range of achievement that is generally low.
- the instruments assessed performance on three types of per cent problems and displays showing results on the enabling objectives should be included.

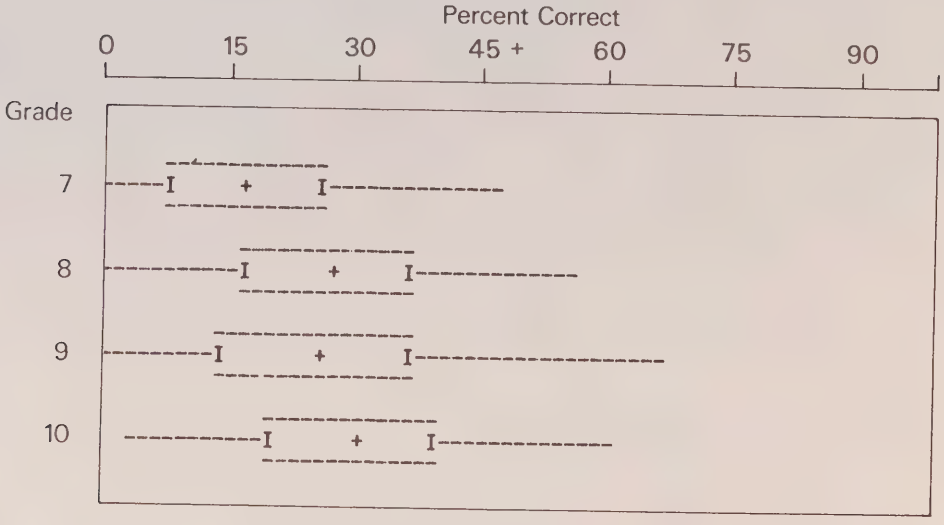
PERCENTAGES

Terminal Objectives in Topic Group

Converting Percents, Ratios and Decimals (N 018 400; p. 91)

Solving a Variety of Percent Problems (N 019 400; p. 97)

Range of Correct Responses to Topic Group, by Grade:



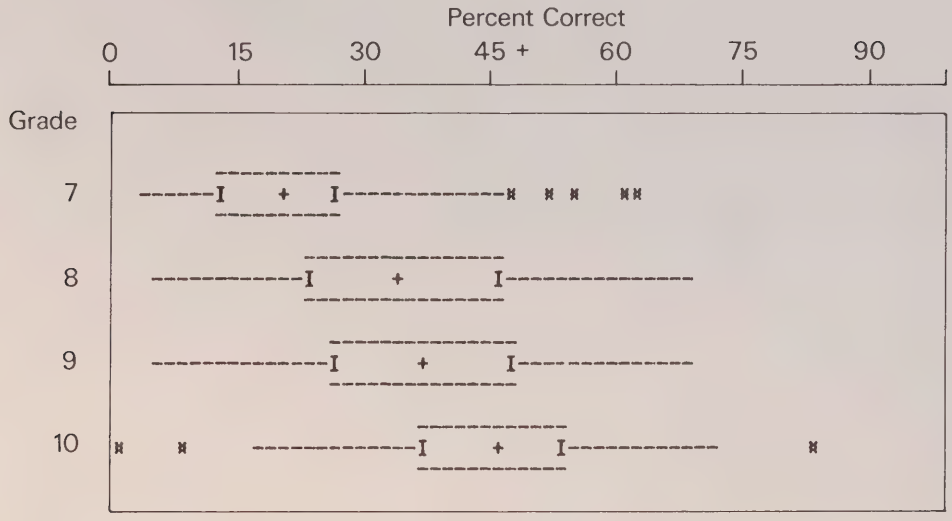
Statistical Summary

Grade Level	No. of Classes	Grade Mean	Grade St. Dev.
7	97	18.6	11.8
8	98	26.8	12.9
9	122	25.6	15.4
10	103	30.4	13.8

PERCENTAGES

Enabling Objectives in Topic Group

Range of Correct Responses to Topic Group, by Grade:



Statistical Summary

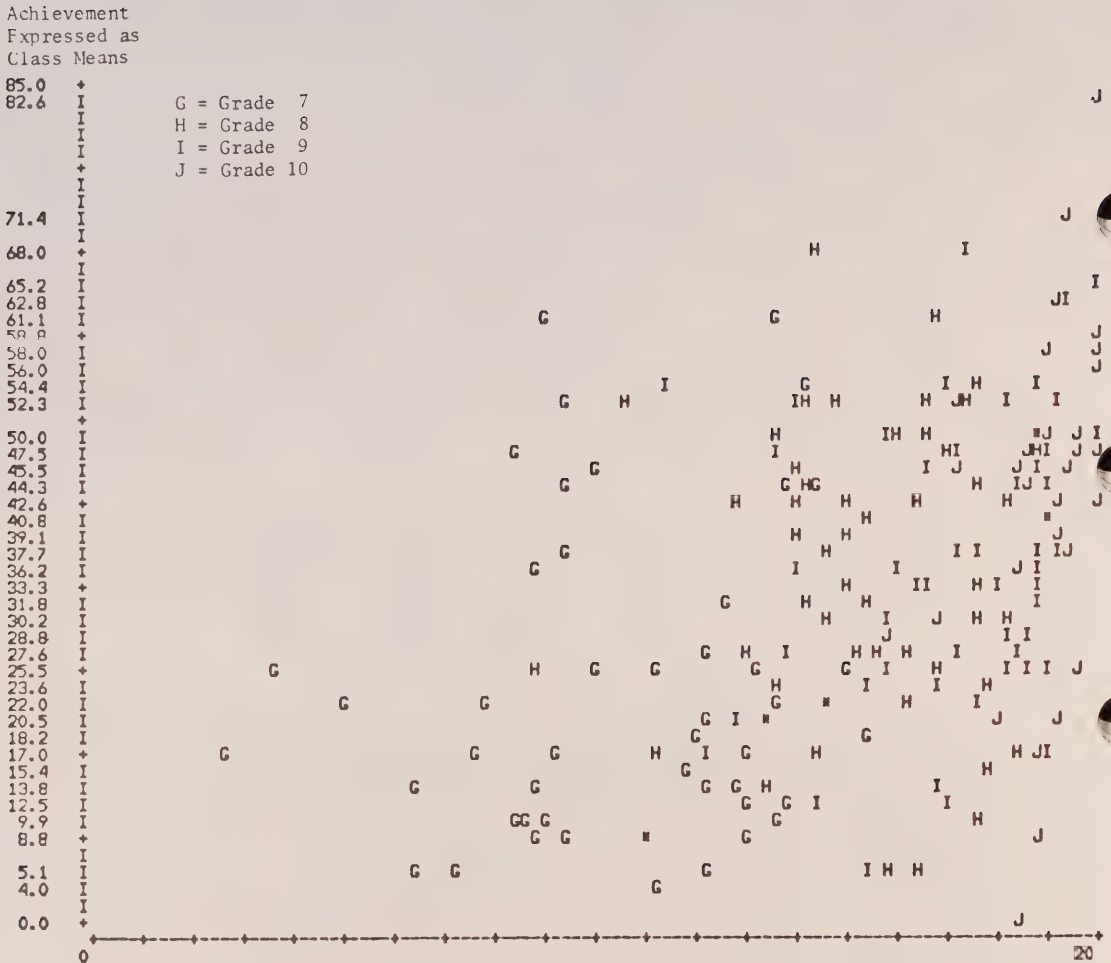
Grade Level	No. of Classes	Grade Mean	Grade St. Dev.
7	53	22.8	14.9
8	54	34.3	14.9
9	54	36.6	14.1
10	33	43.1	17.4

The results displayed in this graph are based on instruments developed from the following objectives in the published pool:

N 018 100: Expressing decimals as per cents and vice versa (p. 87)
N 018 200: Expressing per cents as ratios (p. 89)
N 018 300: Expressing ratios as per cents (p. 90)
N 019 100: Finding what per cent one number is of another number (p. 94)
N 019 200: Finding a per cent of an amount (p. 95)
N 019 300: Finding the whole amount when a part is known (p. 96)

PERCENTAGES

Relationship Between Achievement and Opportunity to Learn:



PRACTICAL PROBLEMS WITH WHOLE NUMBERS

Objective: Simple Problems with Whole Numbers (N 029 300; p. 135)

Instruments Included in the Field Trial:

(1) Jean has $12. He needs $8 more. How much does he need in all?

(2) Mona is selling candy bars for 20 cents each. If she has sold 3 candy bars, how many cents should she have?

(3) A group of 7 people buy a gift which cost 35 dollars. If they split the cost equally, how much must each person pay?

(4) Alicia must pay the bank $2 per month. After 4 months, how much has she paid?

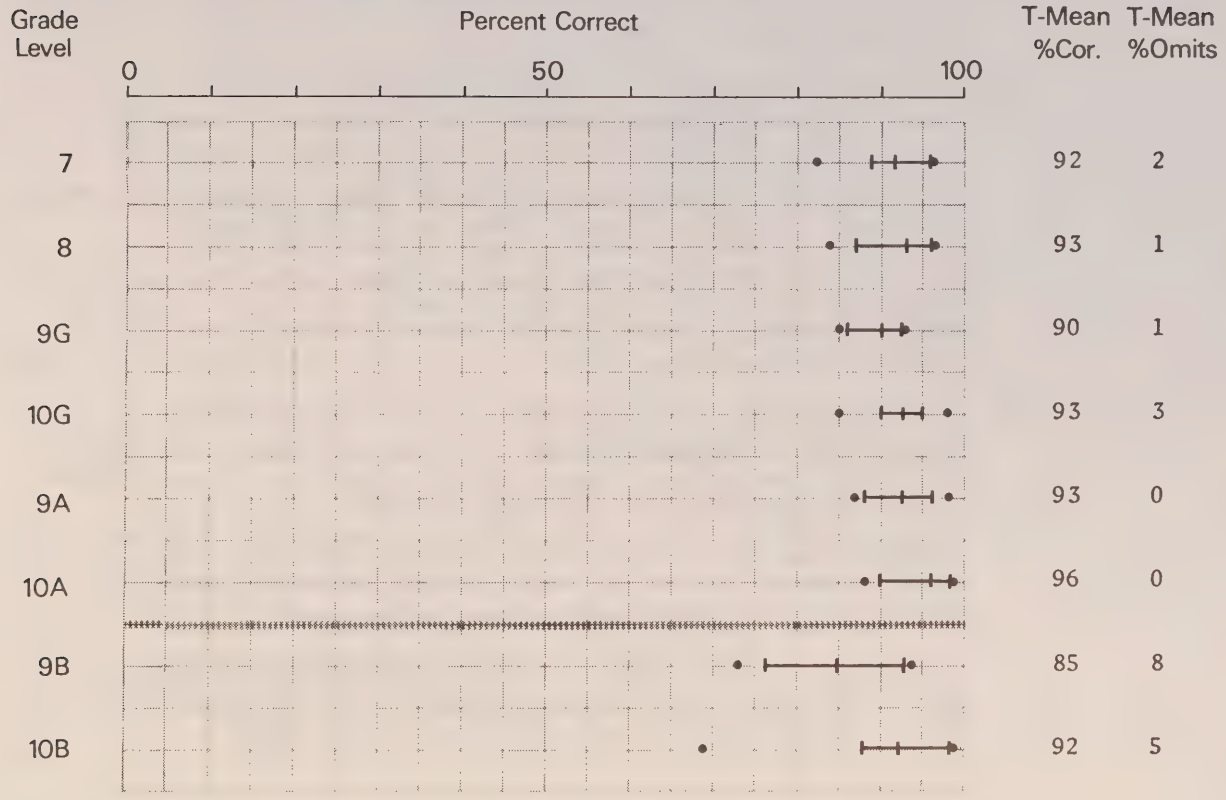
(5) 3 people eat a pizza and split the cost equally. What will it cost each person if the bill is $15?

(6) 6 people are equally splitting a bucket of chicken. If there are 24 pieces of chicken in the bucket, how many will each person get?

PRACTICAL PROBLEMS WITH WHOLE NUMBERS

Objective: Simple Problems with Whole Numbers (N 029 300; p. 135)

Range of Correct Responses to the Six Instruments, by Grade:



- there is a high level of success throughout the Intermediate Division.
- the item form does not provide enough range in difficulty to discriminate between grade levels.

PRACTICAL PROBLEMS WITH WHOLE NUMBERS

Objective: Problems Requiring Multiple Operations (N 029 500; p. 140)

Instruments Included in the Field Trial:

(1) Jill's rent was $100. Later it was raised to $110 per month. The Rent Review Board said that the landlord could only raise the rent to $106 per month. Since Jill paid $100 rent for 2 months, how much did the landlord owe her?*

(2) Ted is stapling together a 3 page newsletter. He has
51 copies of page 1
53 copies of page 2
41 copies of page 3
After he has finished stapling together as many complete newsletters as he can, how many left-over sheets of paper are there?

(3) Mr. Waters is interested in joining a sailing club. The joining fee is $400 and the dues are $30 per month. He must also pay a locker fee of $30 a year. What would it cost him to join the club for one year.

(4) Sally sees a dress at a store for $27, but decides to sew one like it herself. The dress requires 4 m of material at $3 per metre, and other items such as thread cost $3 altogether. How much money does Sally save by making the dress herself?

(5) Carol borrows $6,300 to buy a car. To repay the loan, she pays $300 per month for 2 years. How much more than the original $6,300 will Carol have repaid after the two years?

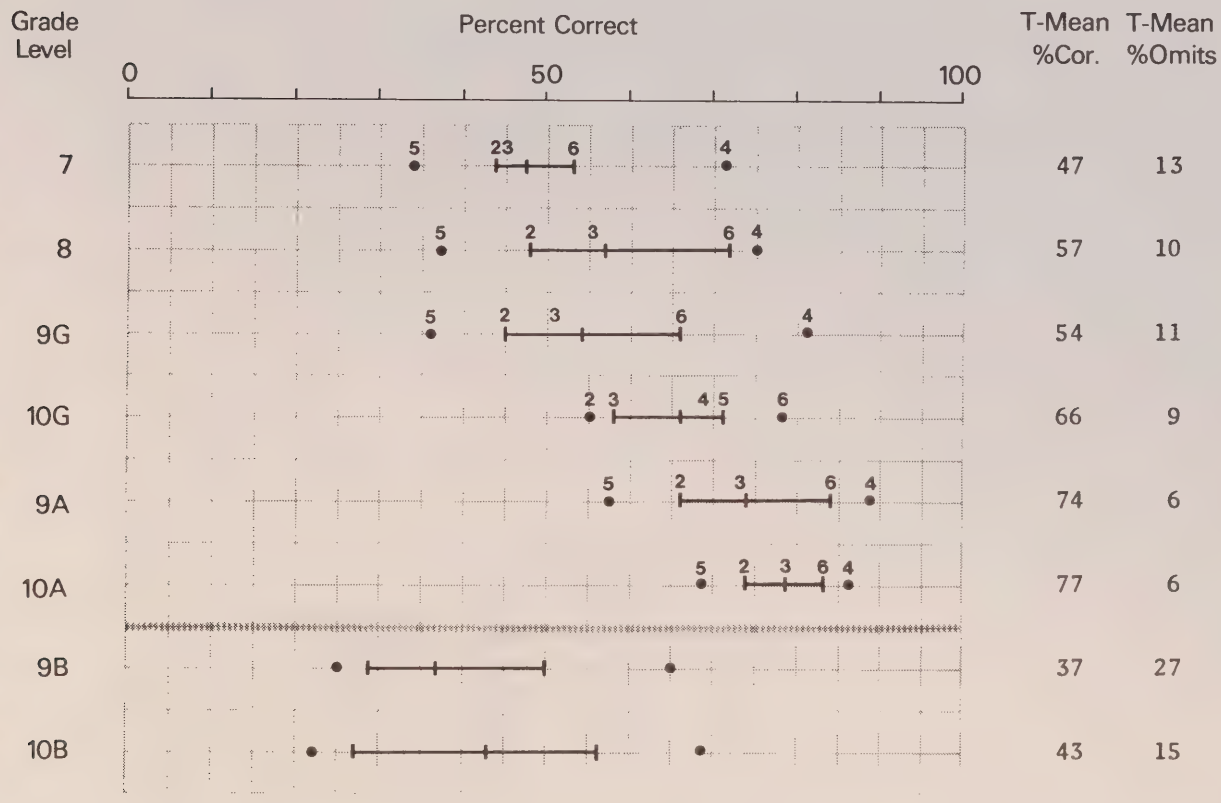
(6) Shawn wants to buy a bicycle that costs $69 but he has only $13 saved. How many weeks will it take him to save the rest of the money, if every week he earns $7 from his paper route?

*This instrument appeared in the booklet with a typographical error. For this reason, it is not included in the analysis of results.

PRACTICAL PROBLEMS WITH WHOLE NUMBERS

Objective: Problems Requiring Multiple Operations (N 029 500; p. 140)

Range of Correct Responses to the Six Instruments, by Grade:



- there is very good progress through the grades. It is encouraging that most Grade 10 Advanced students have little difficulty.
- instruments have good discriminating power over grades.
- difficulty with reading may have contributed to the number of incorrect responses.
- students may have attempted algebraic solutions at Gr. 10 level because that would have been their most recent experience.
- users are advised to include instruments assessing performance on intermediate objectives - i.e., one-step, two-step, multiple step problems.

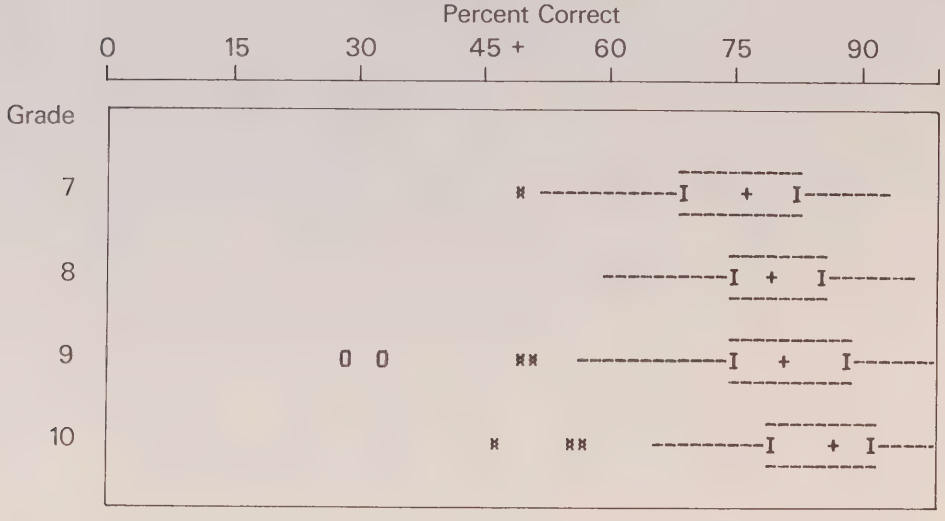
PRACTICAL PROBLEMS WITH WHOLE NUMBERS

Terminal Objectives in Topic Group

Simple Problems with Whole Numbers (N 029 300; p. 135)

Problems Requiring Multiple Operations (N 029 500; p. 140)

Range of Correct Responses to Topic Group, by Grade:



Statistical Summary

Grade Level	No. of Classes	Grade Mean	Grade St. Dev.
7	96	76.3	9.4
8	98	79.9	7.8
9	117	80.8	12.3
10	99	85.3	10.0

PRACTICAL PROBLEMS WITH FRACTIONS AND DECIMALS

Objective: Problems Using Fractions (N 029 700; p. 148)

Instruments Included in the Field Trial:

(1) On Sunday a long distance call costs only $\frac{1}{3}$ of the regular rate. If the regular charge for a call is $12 how much will it cost on a Sunday?

(2) A gas tank holds 48 litres of gasoline. If the tank is $\frac{3}{4}$ full, how many litres of gas does it contain?

(3) Ardith works at a dress shop and pays only $\frac{2}{3}$ of the regular price for dresses there. If a dress cost $21 regularly, how much would Ardith pay for it?

(4) Sam picks $1\frac{1}{2}$ boxes of blueberries and his friend picks 2 boxes. How many boxes did they pick together?

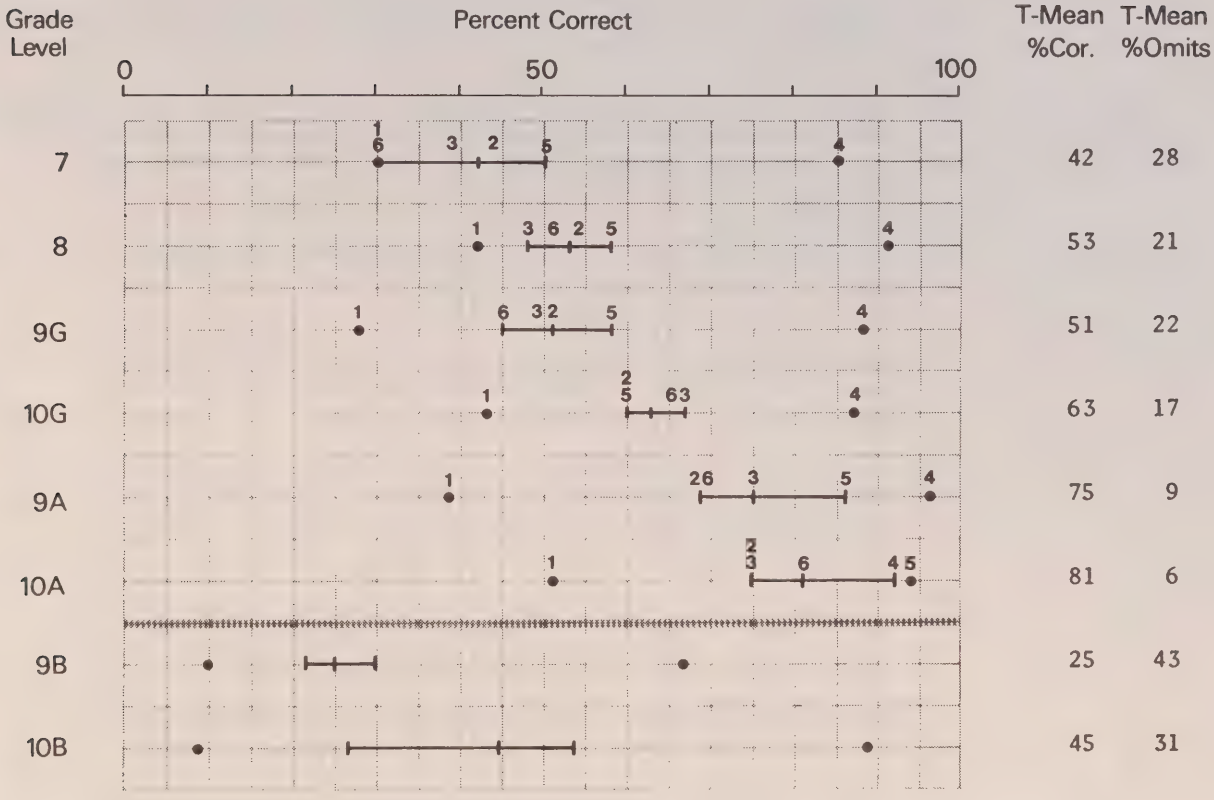
(5) A certain motion needs the votes of $\frac{2}{3}$ of the committee to pass. If there are 9 people on the committee, how many votes are needed to pass the motion?

(6) Ada's part-time job pays $1\frac{1}{2}$ times the usual hourly rate if she works on a holiday. If her usual hourly rate is $8, what is her hourly rate on a holiday?

PRACTICAL PROBLEMS WITH FRACTIONS AND DECIMALS

Objective: Problems Using Fractions (N 029 700; p. 148)

Range of Correct Responses to the Six Instruments, by Grade:



- a comparison of Display 20 with Display 22 suggests students have difficulty with fractions in problem solving situations.
- it is encouraging that progress throughout the Division resulted in an 81% average success rate at the Grade 10 Advanced level.
- the types of problems presented vary widely in difficulty. Students have more difficulty with multiplication than with addition.

PRACTICAL PROBLEMS WITH FRACTIONS AND DECIMALS

Objective: Problems Involving Multiple Operations with Fractions and Decimals (N 029 900; p. 156)

Instruments Included in the Field Trial:

(1) Doug works at a clothing store and received a discount of $\frac{1}{4}$ off anything he buys there. If he buys a shirt and tie, and pays $27.00 for them, how much would a regular customer have to pay for them?

(2) Barry notices a special at the record shop and buys one tape and 5 records. The tape came from a rack marked 2 for $6.99 and the records were 3 for $8.99. How much did Barry pay altogether (to the nearest cent)?

(3) Charlie builds a motorized skate board on 2 consecutive nights after school. He worked the following hours:

$3\frac{1}{2}$ and $1\frac{1}{4}$ hours.

His materials cost $34.30 and he sold the motorized skate board for $48.55. If he considers the profit he made as his "wages", how much did he earn per hour?

(4) Paul works at a clothing store and receives a discount of $\frac{1}{3}$ off anything he buys there. If he buys a $13.00 tie and a $23.00 shirt, how much does he have to pay for them (to the nearest cent)?

(5) It usually takes Melanie about 23 minutes to drive home from work. While sitting in a traffic jam one evening, she notices that after 23 minutes, she is only about $\frac{2}{3}$ of the way home. If she keeps up at that rate, how long will the entire trip home take (to the nearest minute)?

(6) Last spring the average monthly rainfall set a new record. The following figures were obtained.

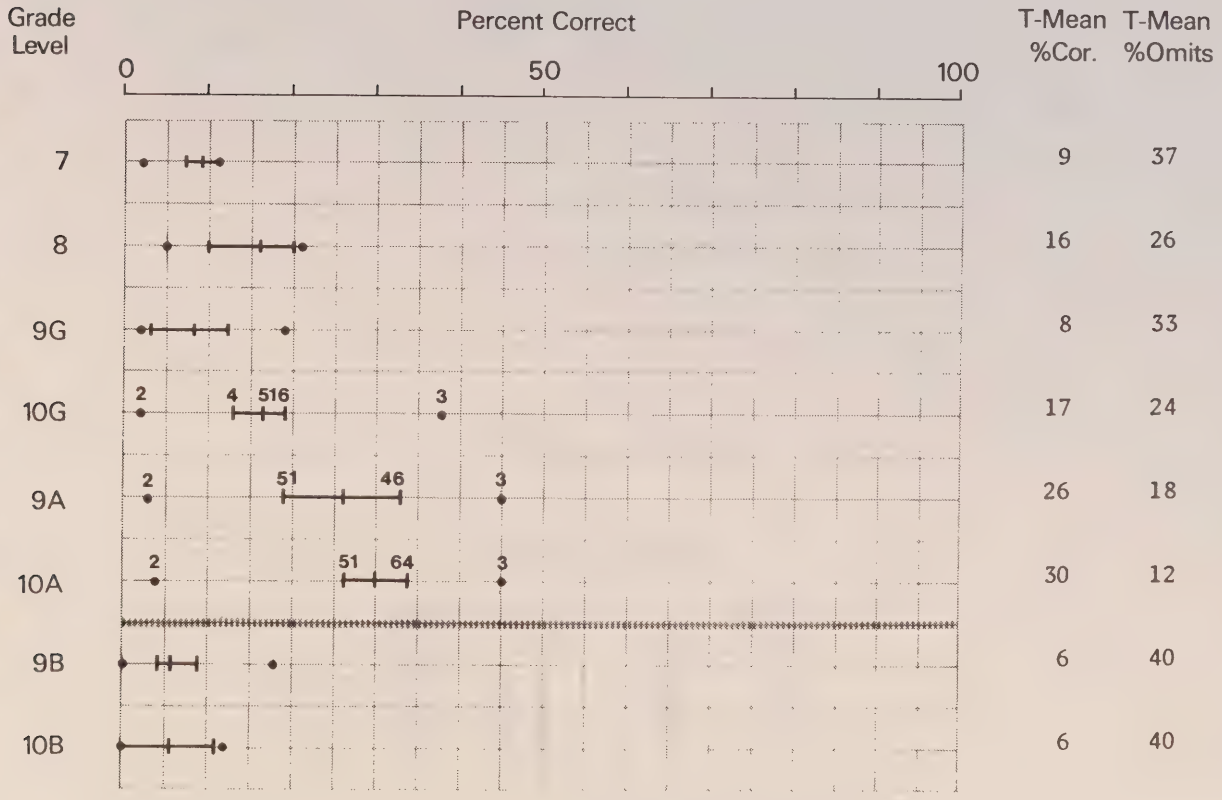
April - 10.22 cm
May - 20.01 cm
June - 24.03 cm

What was the monthly average for those 3 months last spring (to 2 decimal places)?

PRACTICAL PROBLEMS WITH FRACTIONS AND DECIMALS

Objective: Problems Involving Multiple Operations with Fractions and Decimals (N 029 900; p. 156)

Range of Correct Responses to the Six Instruments, by Grade:



- the poor performance of students on these instruments may be explained by the number of sub skills involved.

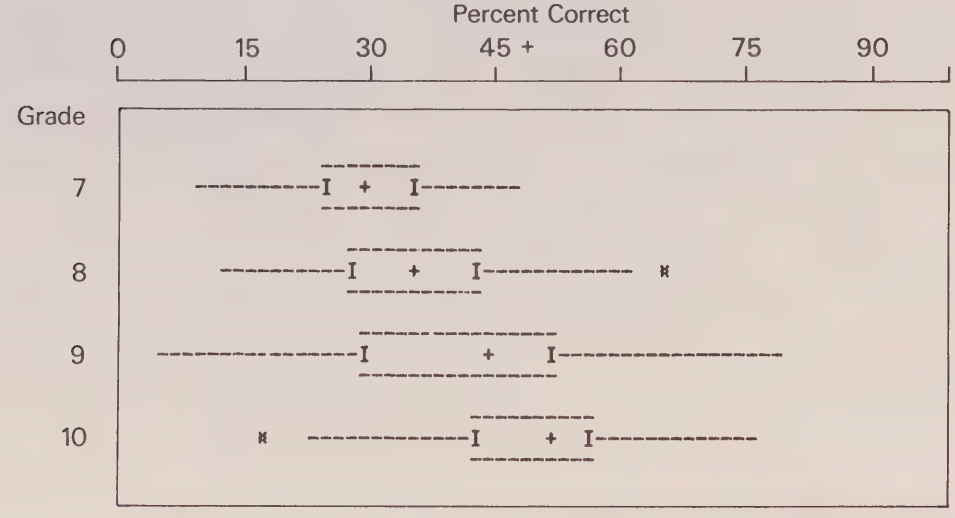
PRACTICAL PROBLEMS WITH FRACTIONS AND DECIMALS

Terminal Objectives in Topic Group

Problems Using Fractions (N 029 700; p. 148)

Problems Involving Multiple Operations with Fractions and Decimals (N 029 900; p. 156)

Range of Correct Responses to Topic Group, by Grade:



Statistical Summary

Grade Level	No. of Classes	Grade Mean	Grade St. Dev.
7	70	28.6	9.5
8	67	35.4	11.2
9	65	42.0	15.5
10	45	49.9	13.1

OPERATIONS WITH INTEGERS

Objective: Adding and Subtracting Integers (N 021 200; p. 106)

Instruments Included in the Field Trial:

(1) $-10 - 17 + 29 - 9 =$

(2) $-10 - 20 - (-35) =$

(3) $1 - (-24) - 30 - (-3) =$

(4) $-11 + 21 - (-34) =$

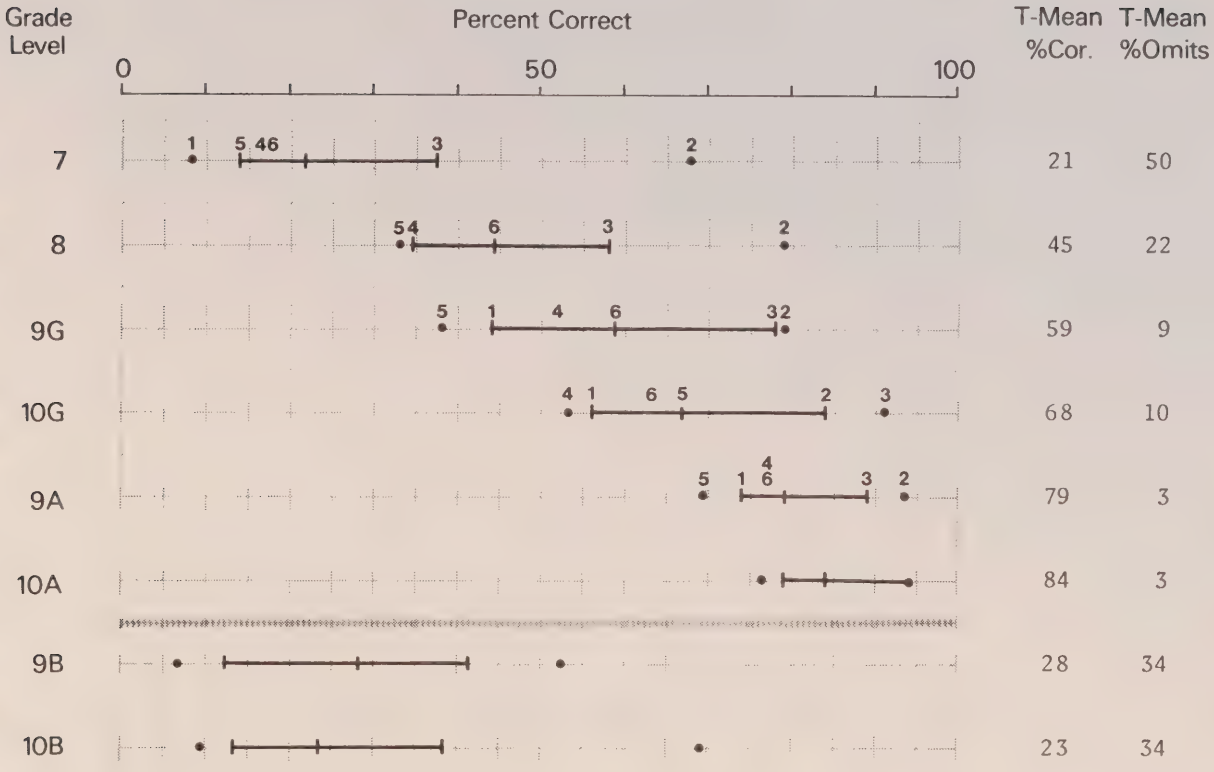
(5) $-14 + 21 - (-26) + 4 =$

(6) $14 - 17 + 30 =$

OPERATIONS WITH INTEGERS

Objective: Adding Integers (N 020 300; p. 102)

Range of Correct Responses to the Six Instruments, by Grade:



- the results reveal the impact teaching has with respect to operations with integers.
- there is remarkably good progress throughout the grades.
- basic level programs obviously do not deal with these topics.
- students had no more difficulty with subtraction than with addition.
- success of students may be related to teaching strategies as reported in the teacher/process questionnaires.
- integers are done so well that less time can be spent on these topics in favour of problem solving.

OPERATIONS WITH INTEGERS

Objective: Adding Integers (N 020 300; p. 102)

Instruments Included in the Field Trial:

(1) $17 + (-6) + (-27) =$

(2) $42 + 45 + (-42) =$

(3) $-1 + 4 + 19 =$

(4) $-30 + 3 + (-24) =$

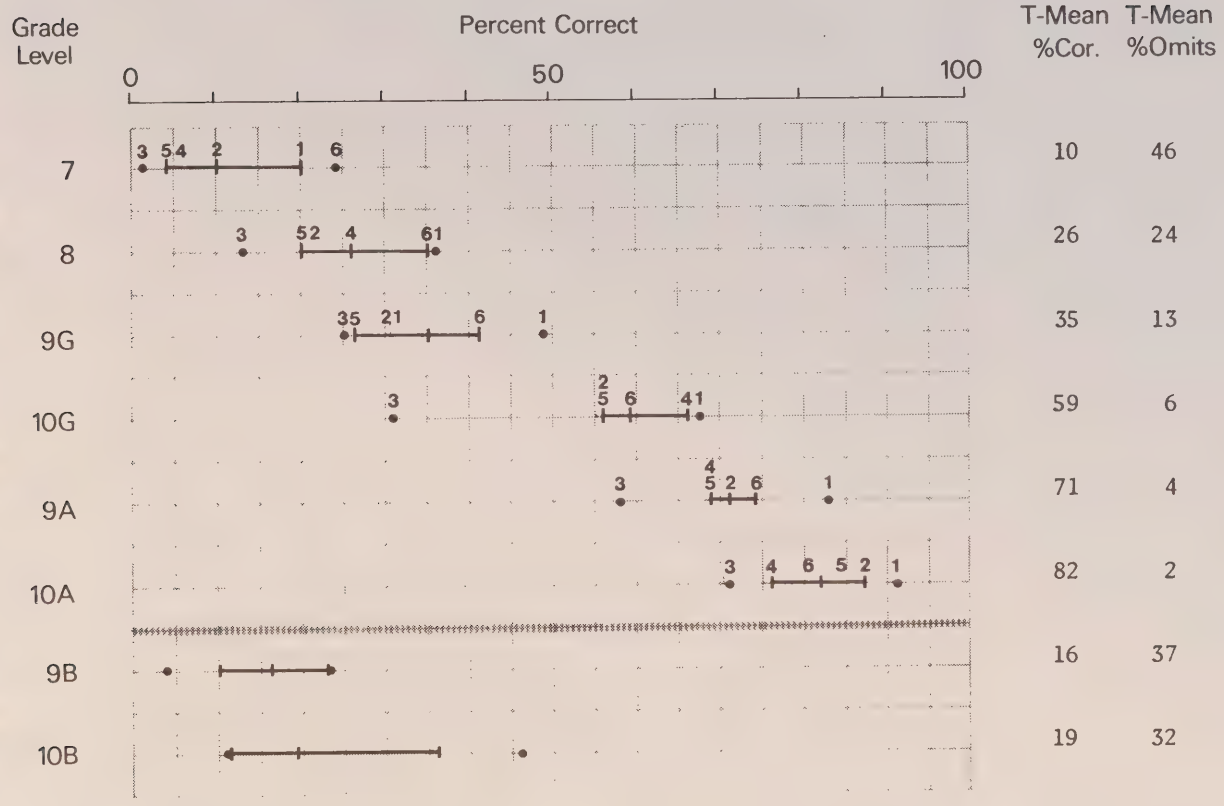
(5) $-15 + (-18) + 21 =$

(6) $-26 + (-27) + (-2) =$

OPERATIONS WITH INTEGERS

Objective: Adding and Subtracting Integers (N 021 200; p. 106)

Range of Correct Responses to the Six Instruments, by Grade:



- the results reveal the impact teaching has with respect to operations with integers.
- there is remarkably good progress throughout the grades.
- basic level programs obviously do not deal with these topics.
- students had no more difficulty with subtraction than with addition.
- success of students may be related to teaching strategies as reported in the teacher/process questionnaires.
- integers are done so well that less time can be spent on these topics in favour of problem solving.

OPERATIONS WITH INTEGERS

Objective: Multiplying Integers (N 022 100; p. 108)

Instruments Included in the Field Trial:

(1) $0 \times -2 \times 10 =$

(2) $8 \times -8 =$

(3) $-4 \times -5 \times -7 =$

(4) $5 \times 0 =$

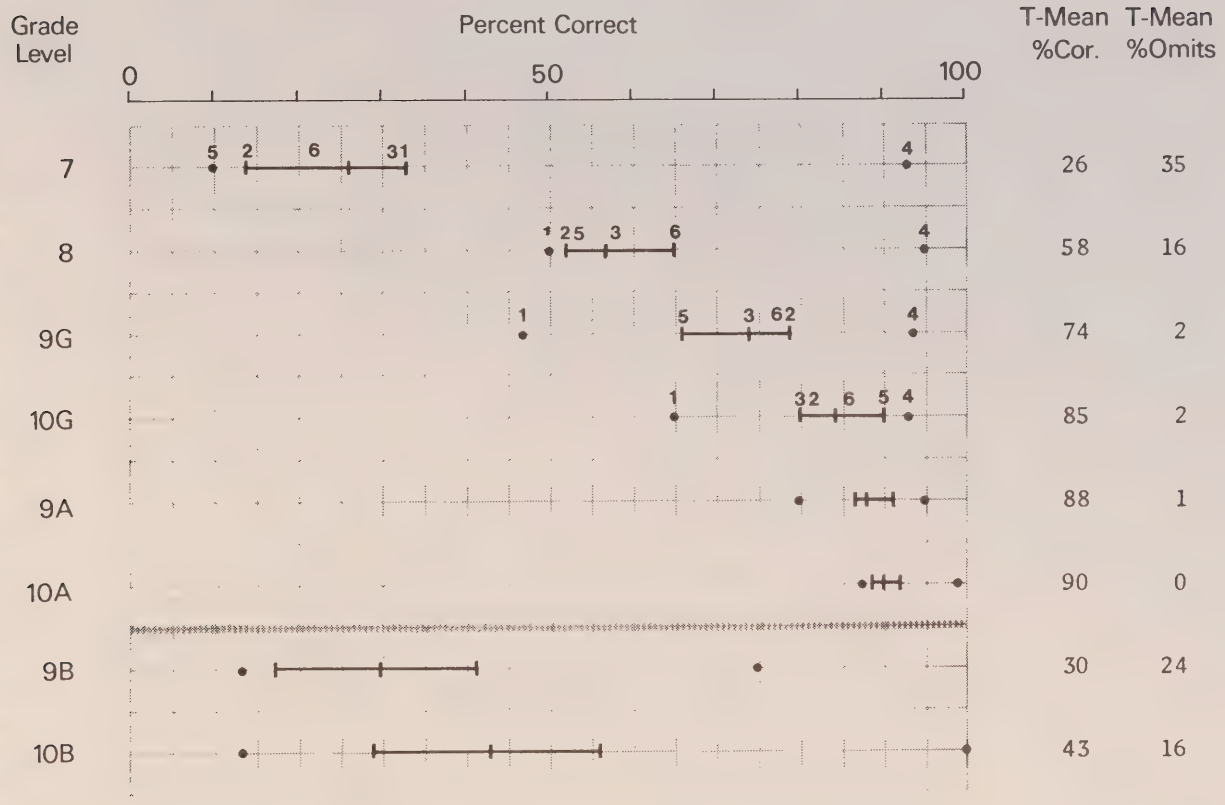
(5) $7 \times -5 \times 2 =$

(6) $-2 \times 7 =$

OPERATIONS WITH INTEGERS

Objective: Multiplying Integers (N 022 100; p. 108)

Range of Correct Responses to the Six Instruments, by Grade:



- the results reveal the impact teaching has with respect to operations with integers.
- there is remarkably good progress throughout the grades.
- basic level programs obviously do not deal with these topics.
- students had no more difficulty with subtraction than with addition.
- Success of students may be related to teaching strategies as reported in the teacher/process questionnaires.
- integers are done so well that less time can be spent on these topics in favour of problem solving.

Objective: Dividing Integers (N 023 100; p. 110)

Instruments Included in the Field Trial:

(1) Express in simplest form

$$\frac{88}{-11} =$$

(2) Express in simplest form

$$\frac{-120}{-12} =$$

(3) Express in simplest form

$$\frac{40}{-10} =$$

(4) Express in simplest form

$$\frac{-1}{1} =$$

(5) Express in simplest form

$$\frac{-36}{6} =$$

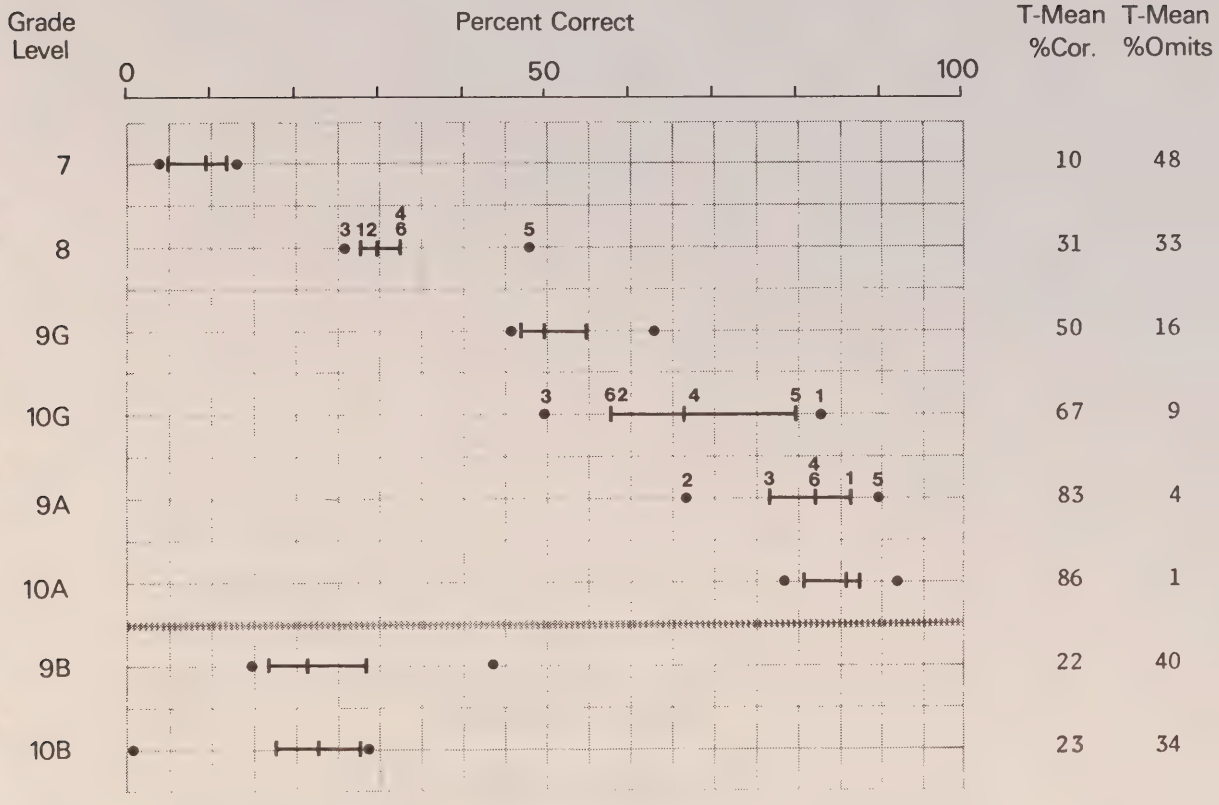
(6) Express in simplest form

$$\frac{9}{-3} =$$

OPERATIONS WITH INTEGERS

Objective: Dividing Integers (N 023 100; p. 110)

Range of Correct Responses to the Six Instruments, by Grade:



- the results reveal the impact teaching has with respect to operations with integers.
- there is remarkably good progress throughout the grades.
- basic level programs obviously do not deal with these topics.
- students had no more difficulty with subtraction than with addition.
- success of students may be related to teaching strategies as reported in the teacher/process questionnaires.
- integers are done so well that less time can be spent on these topics in favour of problem solving.

OPERATIONS WITH INTEGERS

Terminal Objectives in Topic Group

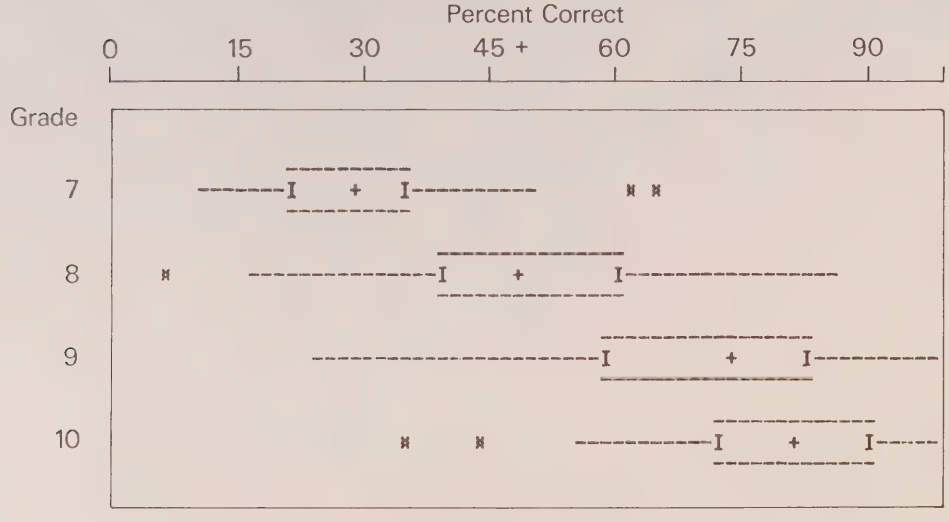
Adding Integers (N 020 300; p. 102)

Adding and Subtracting Integers (N 021 200; p. 106)

Multiplying Integers (N 022 100; p. 108)

Dividing Integers (N 023 100; p. 110)

Range of Correct Responses to Topic Group, by Grade:



Statistical Summary

Grade Level	No. of Classes	Grade Mean	Grade St. Dev.
7	91	29.3	10.2
8	94	49.9	15.8
9	102	70.3	16.9
10	90	80.4	12.2

OPERATIONS WITH POSITIVE AND NEGATIVE NUMBERS IN FRACTION FORM

Objective: Multiplying Rationals (Fraction Form) (N 024 200; p. 113)

Instruments Included in the Field Trial:

(1) Multiply and reduce your answer to lowest terms

$$-3\frac{3}{5} \times \frac{3}{-4} =$$

(2) Multiply and reduce your answer to lowest terms

$$\frac{7}{-9} \times (-9) =$$

(3) Multiply and reduce your answer to lowest terms

$$\frac{-2}{-9} \times \left(\frac{-4}{7}\right) =$$

(4) Multiply and reduce your answer to lowest terms

$$-2\frac{1}{5} \times \left(\frac{-1}{-3}\right) =$$

(5) Multiply and reduce your answer to lowest terms

$$\frac{-1}{-9} \times (-27) =$$

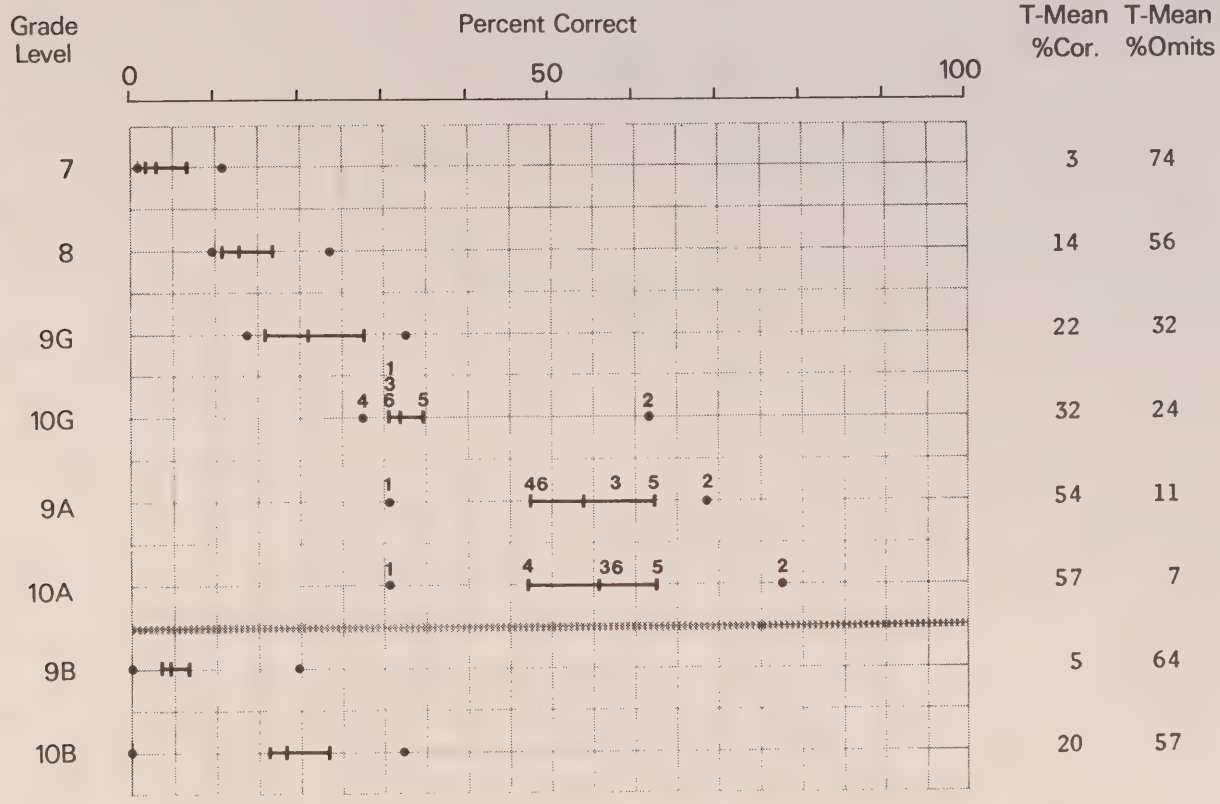
(6) Multiply and reduce your answer to lowest terms

$$\frac{-2}{-8} \times \left(\frac{-6}{7}\right) =$$

OPERATIONS WITH POSITIVE AND NEGATIVE NUMBERS IN FRACTION FORM

Objective: Multiplying Rationals (Fraction Form)(N 024 200; p. 113)

Range of Correct Responses to the Six Instruments, by Grade:



- the progress of students demonstrates the emphasis this topic receives at different grade levels.
- difficulties experienced by the students are consistent with their overall difficulty with fractions.
- this is not a topic requiring increased emphasis.

OPERATIONS WITH POSITIVE AND NEGATIVE NUMBERS IN FRACTION FORM

Objective: Dividing Rationals (Fraction Form) (N 025 100; p. 115)

Instruments Included in the Field Trial:

(1) Divide and reduce your answer to lowest terms

$$\frac{-1\frac{3}{4}}{1\frac{2}{3}} =$$

(2) Divide and reduce your answer to lowest terms

$$\frac{\frac{-1}{-3}}{\frac{3}{7}} =$$

(3) Divide and reduce your answer to lowest terms

$$\frac{-3\frac{1}{4}}{4} =$$

(4) Divide and reduce your answer to lowest terms

$$\frac{2\frac{1}{3}}{\frac{3}{4}} =$$

(5) Divide and reduce your answer to lowest terms

$$\frac{-1\frac{3}{4}}{2\frac{1}{3}} =$$

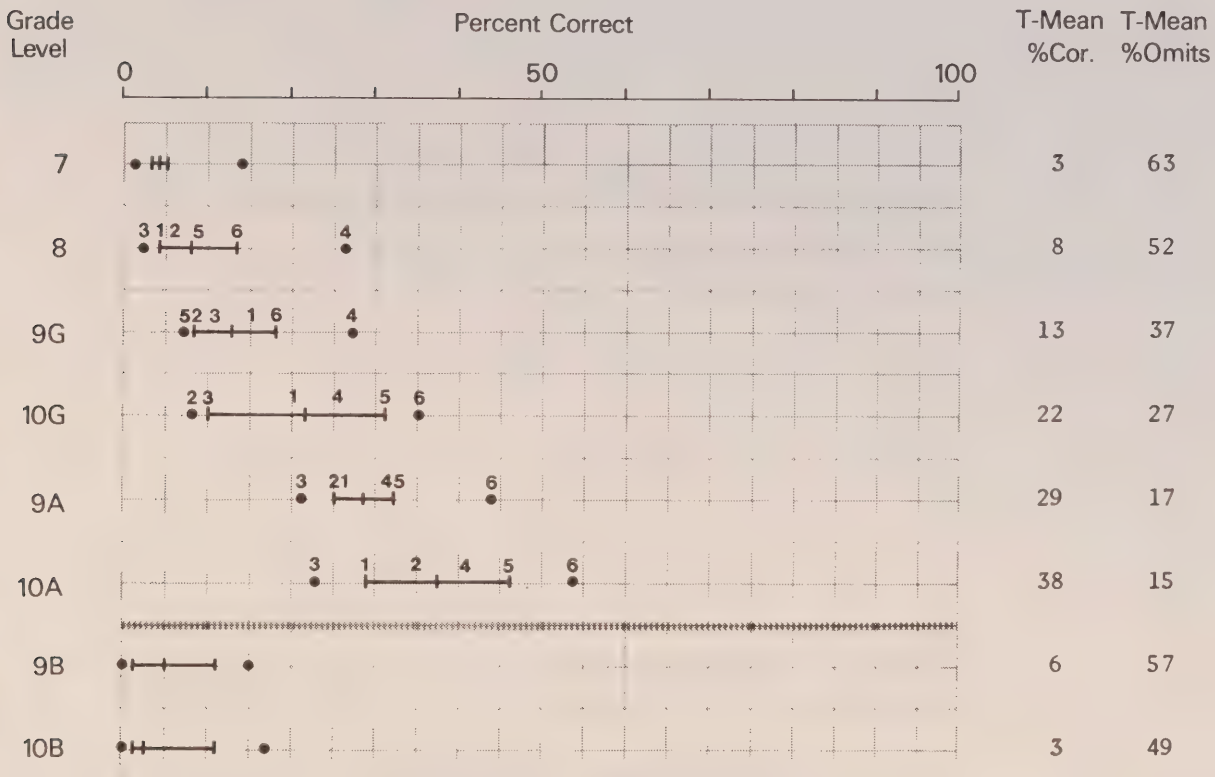
(6) Divide and reduce your answer to lowest terms

$$\frac{\frac{-4}{7}}{\frac{-2}{7}} =$$

OPERATIONS WITH POSITIVE AND NEGATIVE NUMBERS IN FRACTION FORM

Objective: Dividing Rationals (Fraction Form) (N 025 100; p. 115)

Range of Correct Responses to the Six Instruments, by Grade:



- the progress of students demonstrates the emphasis this topic receives at different grade levels.
- difficulties experienced by the students are consistent with their overall difficulty with fractions.
- this is not a topic requiring increased emphasis.

OPERATIONS WITH POSITIVE AND NEGATIVE NUMBERS IN FRACTION FORM

Objective: Adding and Subtracting Rationals (Fraction Form)

(N 026 300; p. 120)

Instruments Included in the Field Trial:

(1) Add and reduce your answer to lowest terms

$\frac{-3}{-4} + 1\frac{3}{6} =$

(2) Subtract and reduce your answer to lowest terms

$2\frac{5}{6} - (\frac{-2}{-5}) =$

(3) Add and reduce your answer to lowest terms

$\frac{-1}{5} + 2\frac{3}{4} =$

(4) Subtract and reduce your answer to lowest terms

$2\frac{1}{6} - \frac{3}{4} =$

(5) Add and reduce your answer to lowest terms

$\frac{5}{6} + 1\frac{1}{4} =$

(6) Subtract and reduce your answer to lowest terms

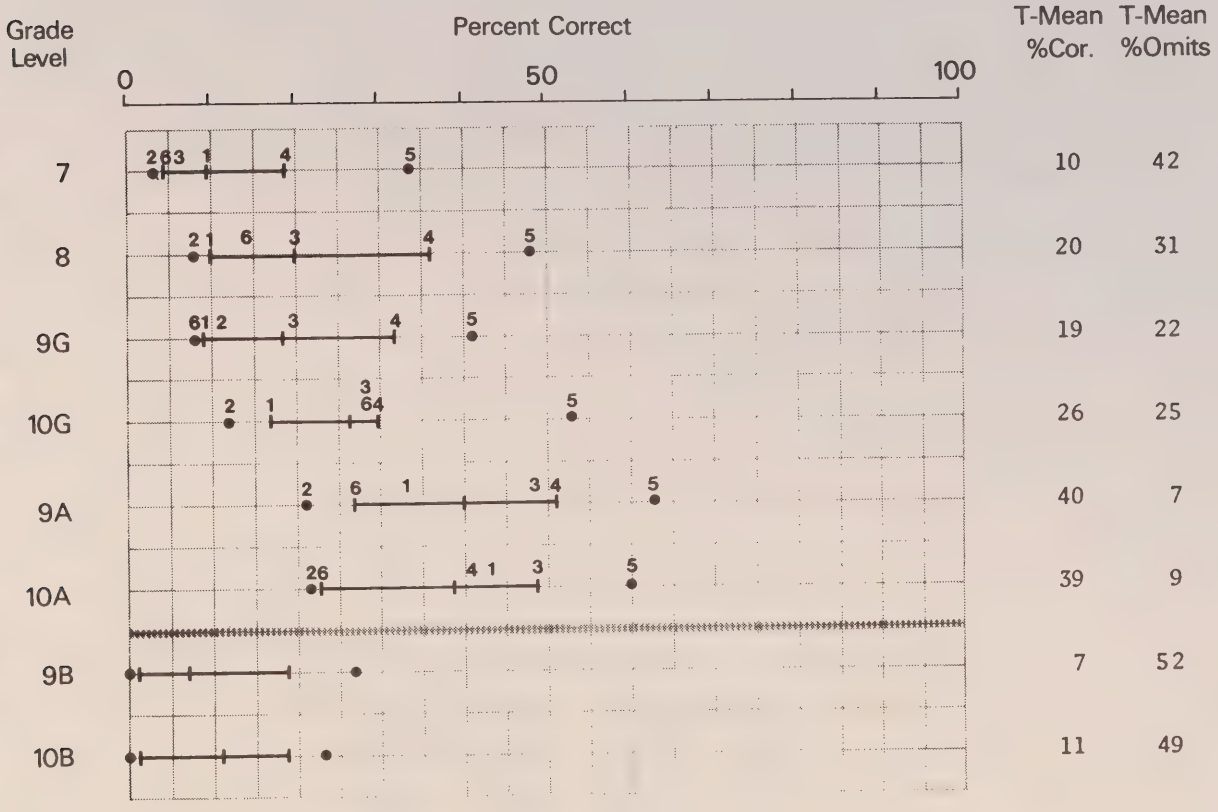
$-1\frac{3}{4} - (\frac{-3}{5}) =$

OPERATIONS WITH POSITIVE AND NEGATIVE NUMBERS IN FRACTION FORM

Objective: Adding and Subtracting Rationals (Fraction Form)

(N 026 300; p. 120)

Range of Correct Responses to the Six Instruments, by Grade:



- the progress of students demonstrates the emphasis this topic receives at different grade levels.
- difficulties experienced by the students are consistent with their overall difficulty with fractions.
- this is not a topic requiring increased emphasis.

OPERATIONS WITH POSITIVE AND NEGATIVE NUMBERS IN FRACTION FORM

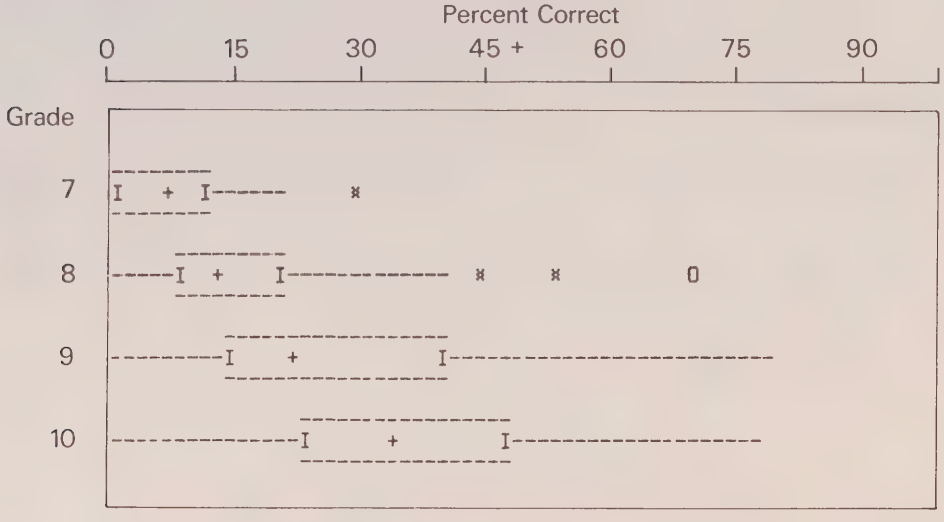
Terminal Objectives in Topic Group

Multiplying Rationals (Fraction Form) (N 024 200; p. 113)

Dividing Rationals (Fraction Form) (N 025 100; p. 115)

Adding and Subtracting Rationals (Fraction Form) (N 026 300; p. 120)

Range of Correct Responses to Topic Group, by Grade:



Statistical Summary

Grade Level	No. of Classes	Grade Mean	Grade St. Dev.
7	96	6.8	6.1
8	98	15.7	11.8
9	118	26.8	18.1
10	98	34.9	17.0

OPERATIONS WITH POSITIVE AND NEGATIVE NUMBERS IN DECIMAL FORM

Objective: Adding and Subtracting Rational Numbers (Decimal Form)

(N 027 300; p. 125)

Instruments Included in the Field Trial:

(1) $2 - 3.1 - (-2) + (-0.9) =$

(2) $1 - 5 + 4.3 - (-3.89) =$

(3) $1 - 5 + (-4.7) - (-1.86) =$

(4) $(-3.83) - (-5) - (-3.8) =$

(5) $(-4.87) - 3.4 - (-4) =$

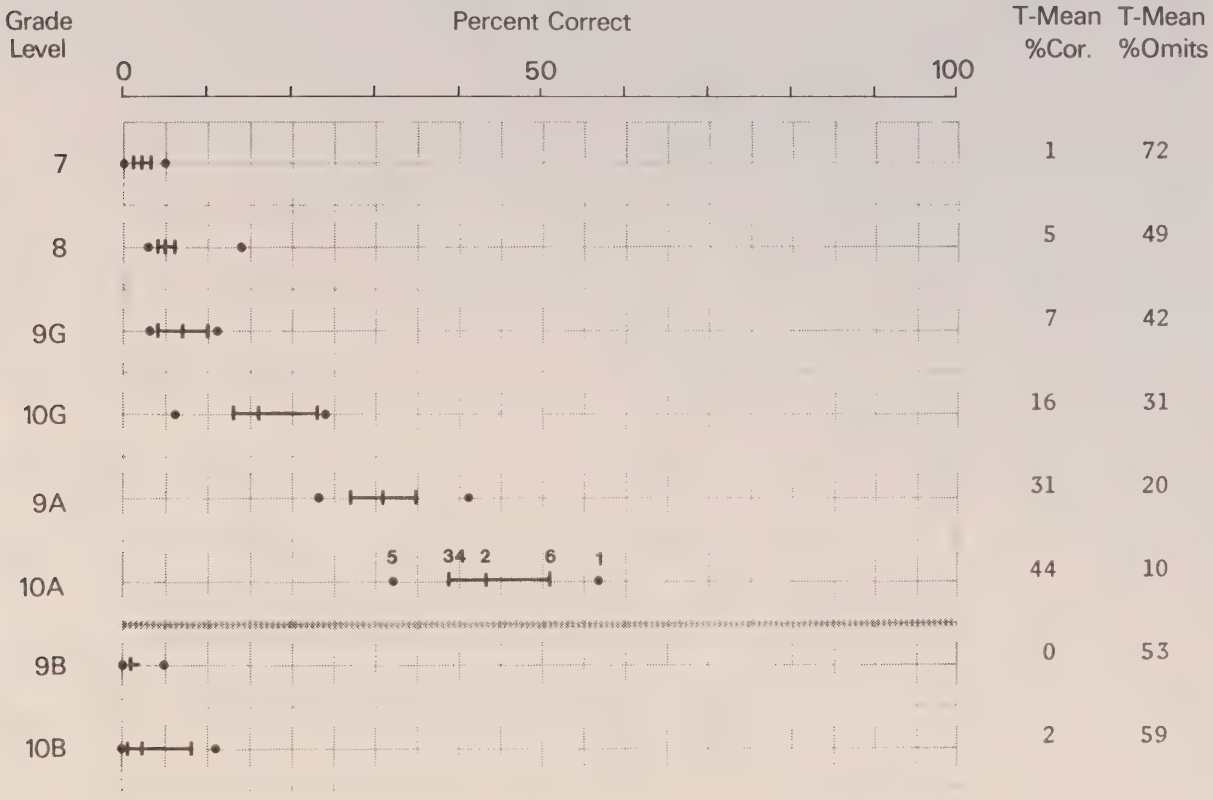
(6) $(-0.36) - 4 - 0.9 =$

OPERATIONS WITH POSITIVE AND NEGATIVE NUMBERS IN DECIMAL FORM

Objective: Adding and Subtracting Rational Numbers (Decimal Form)

(N 027 300; p. 125)

Range of Correct Responses to the Six Instruments, by Grade:



- pupil performance is very low.
- although the objective is not emphasized in the guidelines it is reasonable to expect students at this level to have better skills.

- comparison with Display 9 reveals that student success with a combination of positive and negative decimals is much lower than that with positive decimals.
- teachers should provide more and better opportunities for students to learn these skills.

OPERATIONS WITH POSITIVE AND NEGATIVE NUMBERS IN DECIMAL FORM

Objective: Multiplying and Dividing Rationals (Decimal Form)

(N 028 100; p. 128)

Instruments Included in the Field Trial:

(1) $-6.05 \times -0.04 =$

(2) $\frac{0.244}{-4.3} =$

(Correct to two decimal places)

(3) $-836.9 \times 0.01 =$

(4) $\frac{-9.1008}{0.001} =$

(5) $-0.009 \times -3 =$

(6) $\frac{-0.0822}{0.001} =$

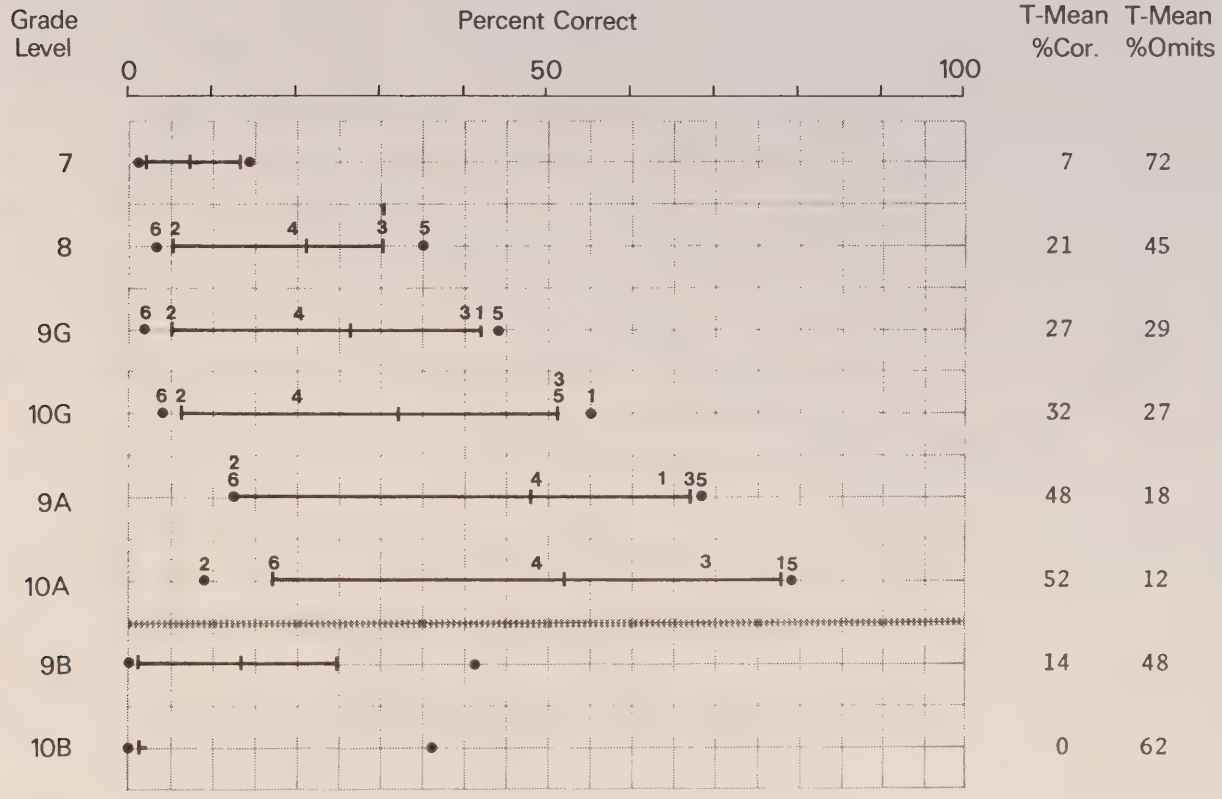
(correct to two decimal places)

OPERATIONS WITH POSITIVE AND NEGATIVE NUMBERS IN DECIMAL FORM

Objective: Multiplying and Dividing Rationals (Decimal Form)

(N 028 100; p. 128)

(Range of Correct Responses to the Six Instruments, by Grade:



- there is progressively better achievement on easier instruments.
- there is extreme diversity in the difficulty levels of instruments generated from this model.
- results are not surprising since this objective is not emphasized in the guideline.
- appropriate objectives for Grades 9 and 10 should be identified separately.
- Intermediate Division Guidelines need to be much more specific on this topic recognizing that calculators will reduce the need for mastering these skills.
- users are advised to select instruments from this model related to specific objectives (Multiplication or Division)

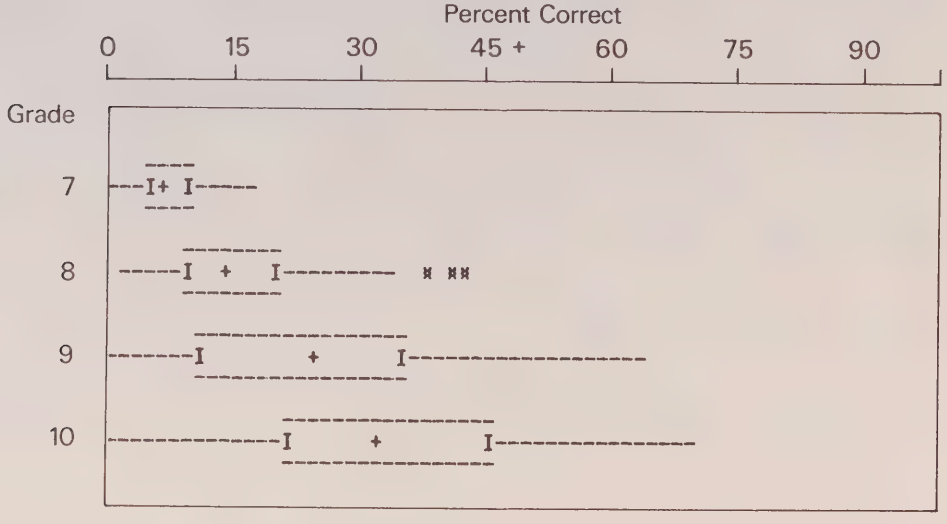
OPERATIONS WITH POSITIVE AND NEGATIVE NUMBERS IN DECIMAL FORM

Terminal Objectives in Topic Group

Adding and Subtracting Rational Numbers (Decimal Form)(N 027 300; p. 125)

Multiplying and Dividing Rationals (Decimal Form) (N 028 100; p. 128)

Range of Correct Responses to Topic Group, by Grade:

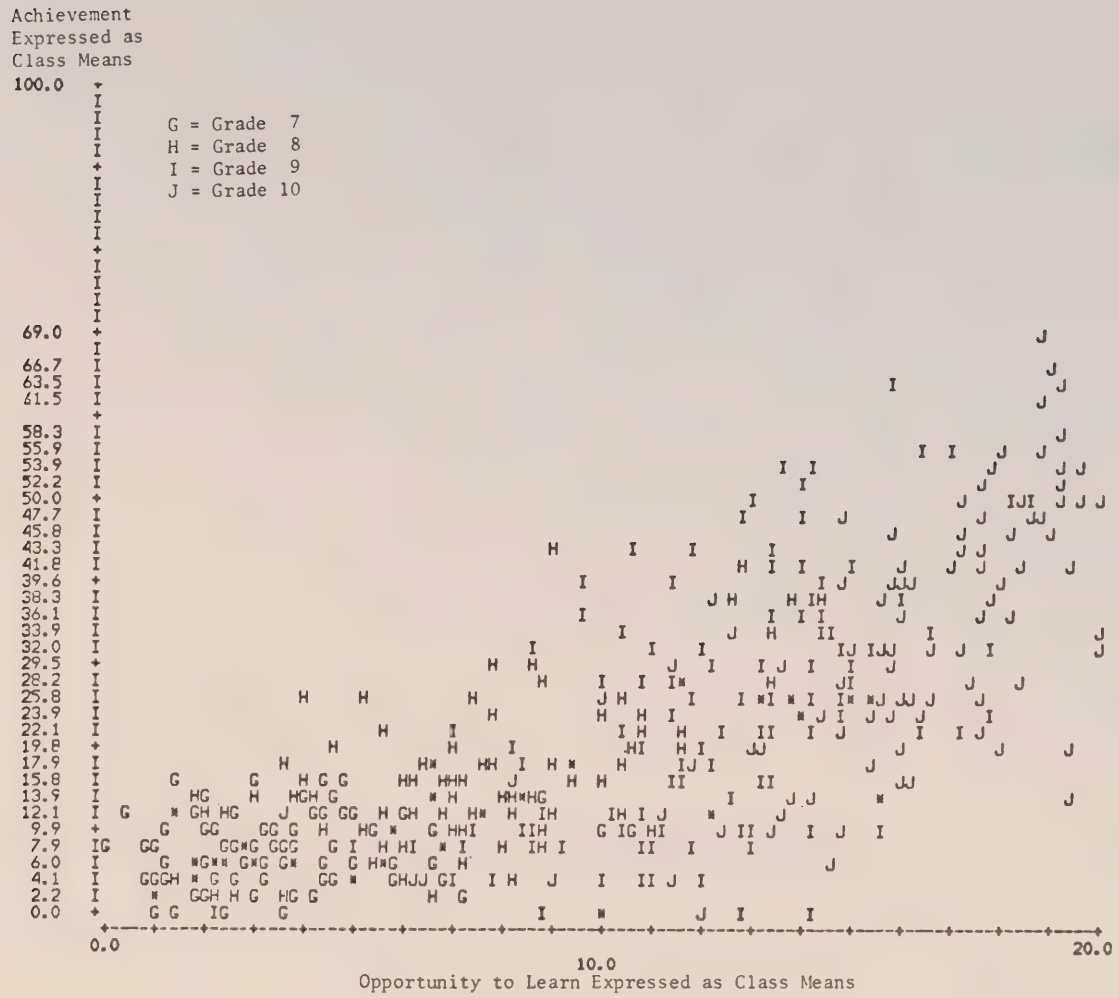


Statistical Summary

Grade Level	No. of Classes	Grade Mean	Grade St. Dev.
7	96	7.5	4.0
8	98	16.0	9.5
9	116	25.0	14.6
10	98	33.1	15.7

OPERATIONS WITH POSITIVE AND NEGATIVE NUMBERS IN DECIMAL FORM

Relationship Between Achievement and Opportunity to Learn:

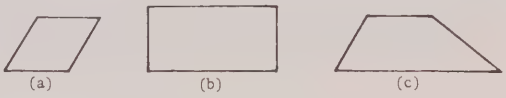


IDENTIFYING SHAPES, FINDING PERIMETERS AND AREAS

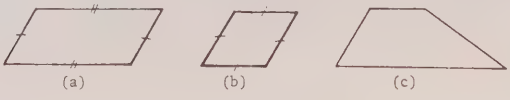
Objective: Identifying Shapes (N 030 100; p. 161)

Instruments Included in the Field Trial:

(1) Which one of the following figures is a rectangle?

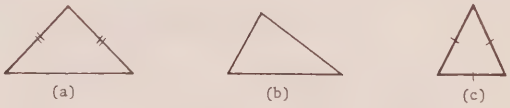


(2) Which one of the following figures is a rhombus?



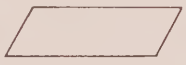
(3) What is the name given to the following figure?

(4) Which one of the following figures is an equilateral triangle?

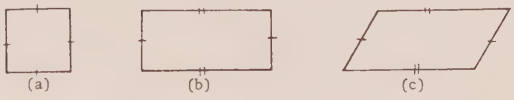


(5) In the following figure, both pairs of opposite sides are parallel.

What is the name given to this figure?



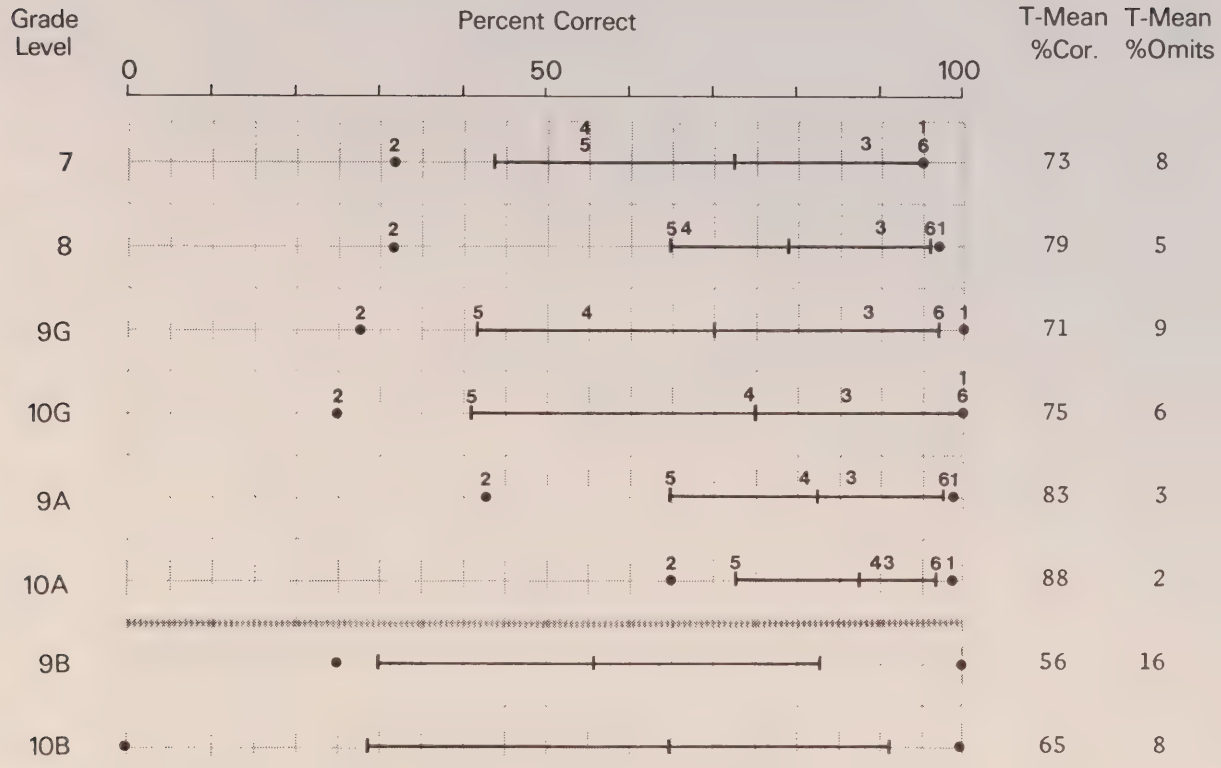
(6) Which of the following figures is a square?



IDENTIFYING SHAPES, FINDING PERIMETERS AND AREAS

Objective: Identifying Shapes (N 030 100; p. 161)

Range of Correct Responses to the Six Instruments, by Grade:



- student performance on this objective is generally good although there is a wide range of success with different figures.
- those instruments emphasizing identification based on definition are more appropriate to the Intermediate Division.

IDENTIFYING SHAPES, FINDING PERIMETERS AND AREAS

Objective: Finding Perimeters (N 030 200; p. 164)

Instruments Included in the Field Trial:

(1) What is the perimeter of a rhombus with one side 3 cm?

(2) What is the perimeter of a rectangle with one side 14 cm and another side 13 cm?

(3) What is the perimeter of a parallelogram with one side 16 cm and another side 9 cm?

(4) What is the perimeter of a rhombus with one side 16 cm?

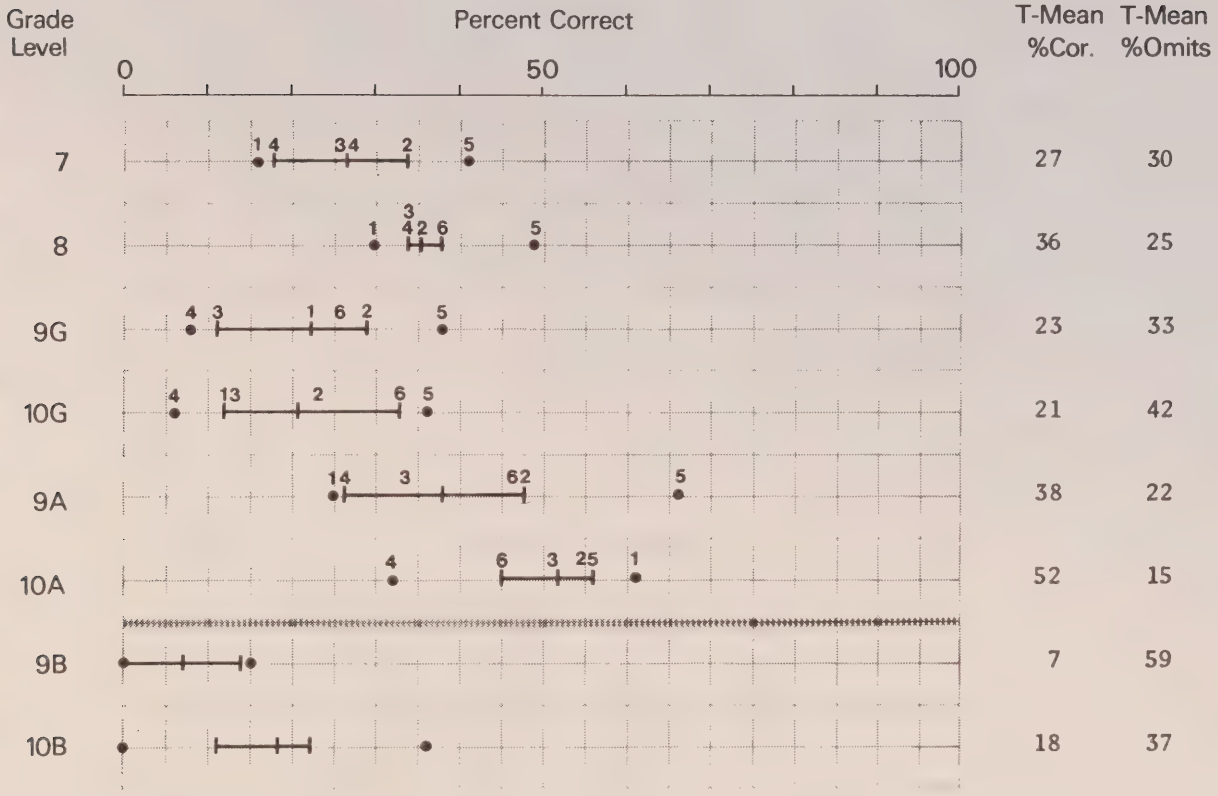
(5) What is the perimeter of an isosceles triangle with two sides 1 cm and third side 2 cm?

(6) What is the perimeter of a rectangle with one side 16 cm and another side 7 cm?

IDENTIFYING SHAPES, FINDING PERIMETERS AND AREAS

Objective: Finding Perimeters (N 030 200; p. 164)

Range of Correct Responses to the Six Instruments, by Grade:



- there is generally low performance on these instruments which may be due to the lack of diagrams.
- the requirement imposed during scoring, that proper units be included may have reduced scores.
- performance indicates the need for more exposure to this kind of problem.

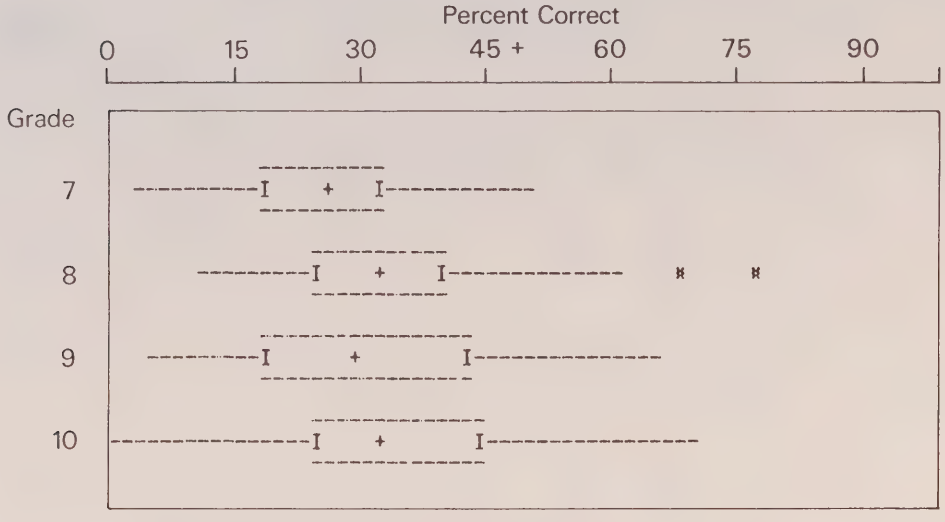
IDENTIFYING SHAPES, FINDING PERIMETERS AND AREAS

Terminal Objectives in Topic Group

Identifying Shapes (N 030 100; p. 161)

Finding Perimeters (N 030 200; p. 164)

Range of Correct Responses to Topic Group, by Grade:



Statistical Summary

Grade Level	No. of Classes	Grade Mean	Grade St. Dev.
7	97	26.3	9.4
8	98	33.3	11.2
9	125	30.8	14.2
10	107	33.3	13.5

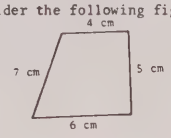
TRAPEZOIDS AND CIRCLES

Objective: Calculating Perimeter and Area of Trapezoid

(Not in published Pool in this form. N 030 600; p. 176, perimeter/ N 030 700; p. 178, area)

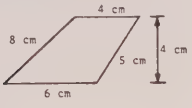
Instruments Included in the Field Trial:

(1) Consider the following figure:



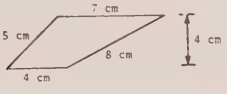
(a) What is the perimeter of the figure (sides 4 cm, 7 cm, 6 cm and 5 cm)?
(b) What is the area of the figure (parallel sides of 4 cm and 6 cm and height of 5 cm)?

(2) Consider the following figure:



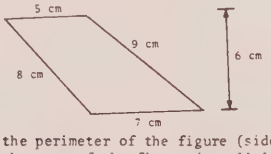
(a) What is the perimeter of the figure (sides 4 cm, 8 cm, 6 cm and 5 cm)?
(b) What is the area of the figure (parallel sides of 4 cm and 6 cm and height of 4 cm)?

(3) Consider the following figure:



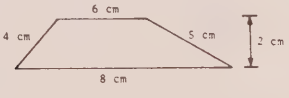
(a) What is the perimeter of the figure (sides 7 cm, 5 cm, 4 cm and 8 cm)?
(b) What is the area of the figure (parallel sides of 7 cm and 4 cm and height of 4 cm)?

(4) Consider the following figure:



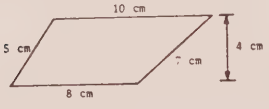
(a) What is the perimeter of the figure (sides 5 cm, 8 cm, 7 cm and 9 cm)?
(b) What is the area of the figure (parallel sides of 5 cm and 7 cm and height of 6 cm)?

(5) Consider the following figure:



(a) What is the perimeter of the figure (sides 4 cm, 6 cm, 6 cm and 14 cm)?
(b) What is the area of the figure (parallel sides of 6 cm and 14 cm and height of 3 cm)?

(6) Consider the following figure:



(a) What is the perimeter of the figure (sides 10 cm, 5 cm, 8 cm and 7 cm)?
(b) What is the area of the figure (parallel sides of 10 cm and 8 cm and height of 4 cm)?

TRAPEZOIDS AND CIRCLES

Objective: Calculating Perimeter and Area of Trapezoid

(Not in published Pool in this form. N 030 600; p. 176, perimeter/ N 030 700; p. 178, area)

Responses to Two-Part Instruments, by Grade:

	Grade			
	7	8	9	10
Both Correct	1	1	3	3
Area Only Correct	0	1	0	1
Perimeter Only Correct	57	63	56	53
Both Incorrect	42	34	36	34
Both Omitted	7	9	36	11

- some students can calculate perimeters but few can find area. The overall results are disappointing.
- the determination of area as the sum of the areas of scalene triangles should be emphasized when formula not known.
- the requirement imposed during scoring, that proper units be included may have reduced scores.
- more time and stress are needed at appropriate grades and levels.

TRAPEZOIDS AND CIRCLES

Objective: Calculating Circumference and Area of a Circle

(Not in published Pool in this form. N 030 800; p. 180, circumference/ N 030 900; p. 182, area)

Instruments Included in the Field Trial:

(1) (a) What is the circumference of a circle of radius of 14 cm (use the value of $\pi = 3.1$)?

(b) What is the area of the circle with radius of 14 cm (use the value of $\pi = 3.1$)?

(2) (a) What is the circumference of a circle of radius of 1 cm (use the value of $\pi = 3.1$)?

(b) What is the area of the circle with radius of 1 cm (use the value of $\pi = 3.1$)?

(3) (a) What is the circumference of a circle of radius of 18 cm (use the value of $\pi = 3.1$)?

(b) What is the area of the circle with radius of 18 cm (use the value of $\pi = 3.1$)?

(4) (a) What is the circumference of a circle with a diameter of 20 cm (use the value of $\pi = 3.1$)?

(b) What is the area of the circle with diameter of 20 cm (use the value of $\pi = 3.1$)?

(5) (a) What is the circumference of a circle with a diameter of 10 cm (use the value of $\pi = 3.1$)?

(b) What is the area of the circle with a diameter of 10 cm (use the value of $\pi = 3.1$)?

(6) (a) What is the circumference of a circle of radius 5 cm (use the value of $\pi = 3.1$)?

(b) What is the area of the circle with radius of 5 cm (use the value of $\pi = 3.1$)?

TRAPEZOIDS AND CIRCLES

Objective: Calculating Circumference and Area of a Circle

(Not in published Pool in this form. N 030 800; p. 180, circumference/ N 030 900; p. 182, area)

Responses to Two-Part Instruments, by Grade:

	Grade			
	7	8	9	10
Both Correct	0	4	5	5
Area Only Correct	1	2	5	4
Circumference Only Correct	2	8	9	9
Both Incorrect	25	47	47	48
Both Omitted	71	37	23	23

- the results on these instruments are disappointing.
- more time and stress are needed at appropriate grades and levels.

TRAPEZOIDS AND CIRCLES

Terminal Objectives in Topic Group

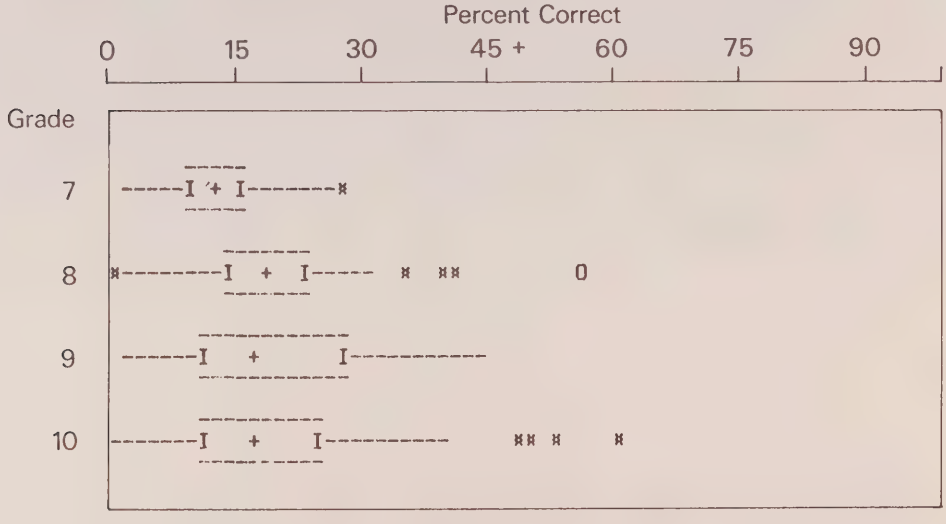
Calculating Perimeter and Area of Trapezoid

(Not in published Pool in this form. N 030 600; p. 176, perimeter/N 030 700; p. 178, area)

Calculating Circumference and Area of a Circle

(Not in published Pool in this form. N 030 800; p. 180, circumference/N 030 900; p. 182, area)

Range of Correct Responses to Topic Group, by Grade:



Statistical Summary

Grade Level	No. of Classes	Grade Mean	Grade St. Dev.
7	97	12.81	5.05
8	98	18.79	8.28
9	122	18.78	10.61
10	104	19.69	11.39

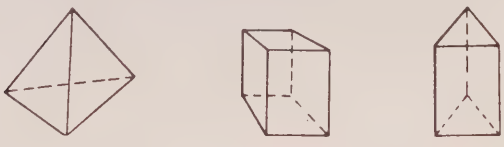
Objective: Identifying Various Prisms and Pyramids (N 031 100; p. 184)

Instruments Included in the Field Trial:

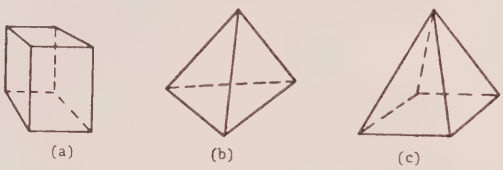
(1) Which of the following figures is a cone?

(2) Name the figure pictured in the following diagram:

(3) Which of the following figures is a triangular prism?

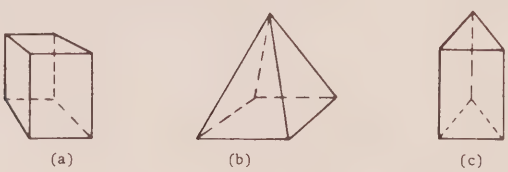


(4) Which of the following figures is a rectangular pyramid?



(5) Name the figure pictured in the following diagram:

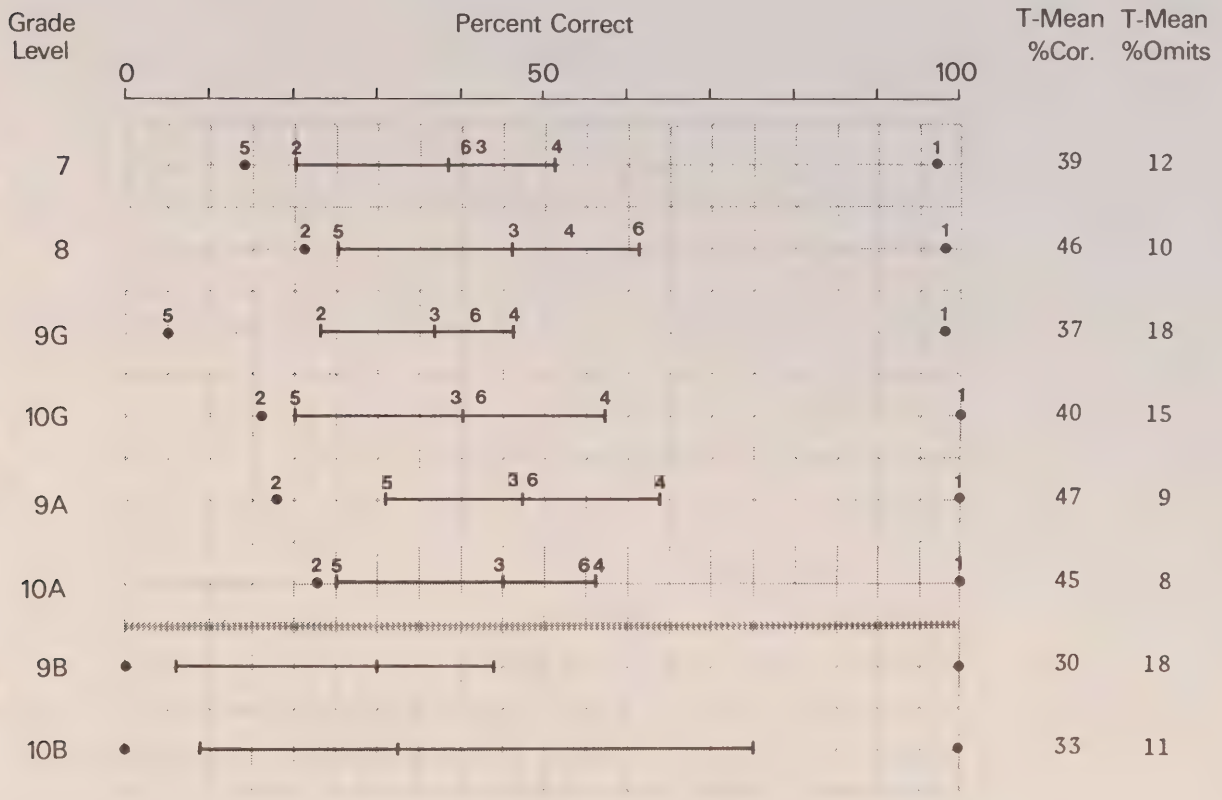
(6) Which of the following figures is a rectangular prism?



IDENTIFYING THREE-DIMENSIONAL SHAPES, FINDING VOLUMES

Objective: - Identifying Various Prisms and Pyramids (N 031 100; p. 184)

Range of Correct Responses to the Six Instruments, by Grade:



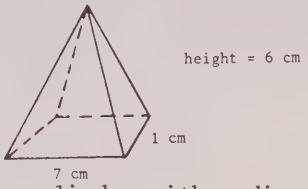
- performance on these instruments is quite low.
- this is a low priority topic for most teachers.
- users are advised to select instruments related to specific objective to be assessed.

IDENTIFYING THREE-DIMENSIONAL SHAPES, FINDING VOLUMES

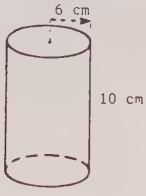
Objective: Identifying Various Prisms and Pyramids (Not in published Pool)

Instruments Included in the Field Trial:

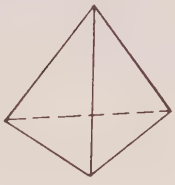
(1) What is the volume of a rectangular pyramid with length of base 7 cm, width of base 1 cm and height 6 cm?



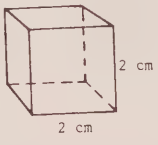
(2) What is the volume of a cylinder with radius of 6 cm and height of 10 cm?



(3) What is the volume of a triangular pyramid with area of base 24 cm^2 and height 5 cm?

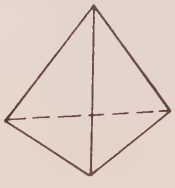


(4) What is the volume of a cube with sides of 2 cm?



(5) What is the volume of a cone with radius of base 1 cm and height 21 cm (use the value of $\pi = 3.1$)?

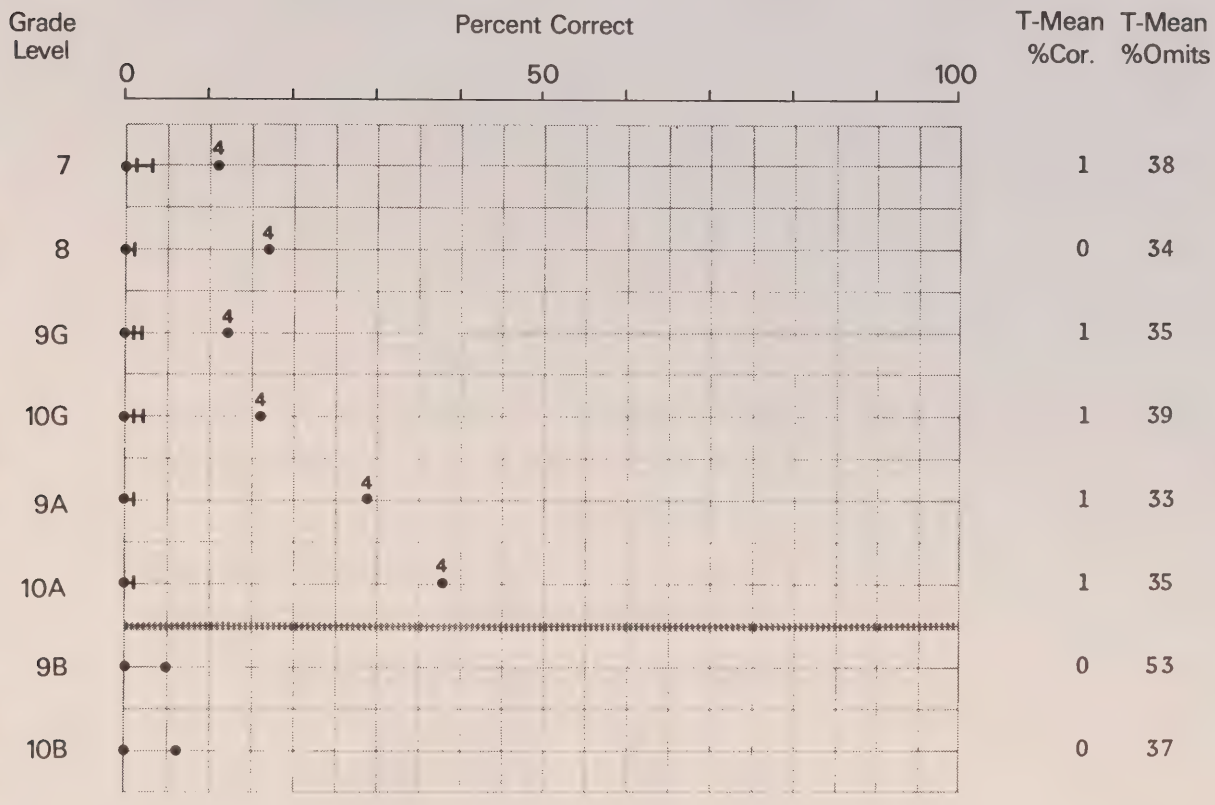
(6) What is the volume of a triangular pyramid with area of base 30 cm^2 height 7 cm?



IDENTIFYING THREE-DIMENSIONAL SHAPES, FINDING VOLUMES

Objective: Determining the Volume of Prisms and Pyramids

(Not in published Pool)



- there is extraordinarily low achievement on this topic.
- some of the shapes are not mentioned in the guideline; (eg. the cylinder is optional in Gr. 8, the pyramid is optional in Gr. 9).
- according to the teacher questionnaire 79% of Gr. 9 teachers do not teach the topic.
- the requirement, imposed during scoring, that proper units be included may have reduced scores.
- teachers should direct more attention to this topic.

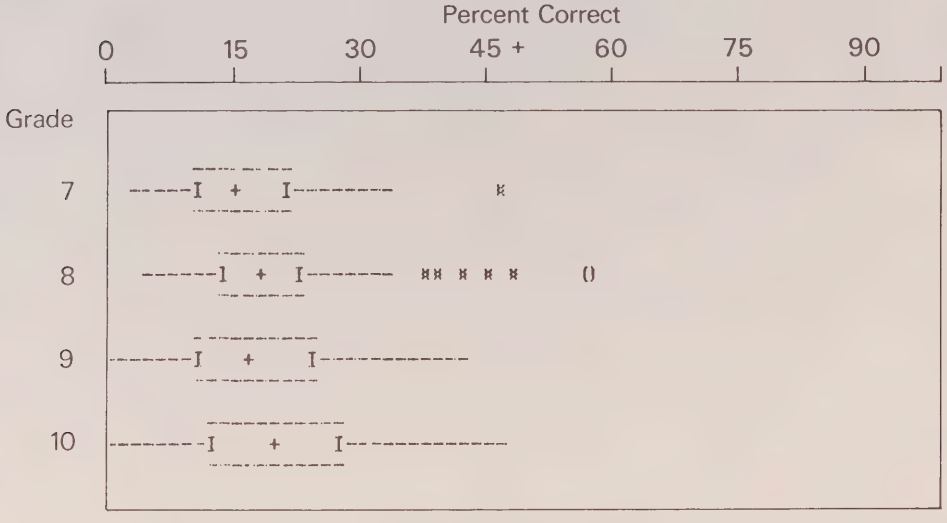
IDENTIFYING THREE-DIMENSIONAL SHAPES, FINDING VOLUMES

Terminal Objectives in Topic Group

Identifying Various Prisms and Pyramids (N 031 100; p. 184)

Determining the Volume of Prisms and Pyramids (Not in published Pool)

Range of Correct Responses to Topic Group, by Grade:



Statistical Summary

Grade Level	No. of Classes	Grade Mean	Grade St. Dev.
7	96	16.83	7.69
8	98	20.49	9.51
9	118	18.52	9.29
10	102	20.99	10.17

ALGEBRA--WORKING WITH FORMULAS

Objective: Converting Word Phrases to Algebraic Phrases

(Not in published Pool)

Instruments Included in the Field Trial:

(1) Using the variable 'a', write an expression which means: "multiply a number by 40, then subtract 24"?

(2) Using the variable 's', write an expression which means: "a number decreased by 41, then multiplied by 34"?

(3) Using the variable 'f', write an expression which means: "3 times the sum of a number and 33"?

(4) Using the variable 'n', write an expression which means: "a number divided by 44, then diminished by 28"?

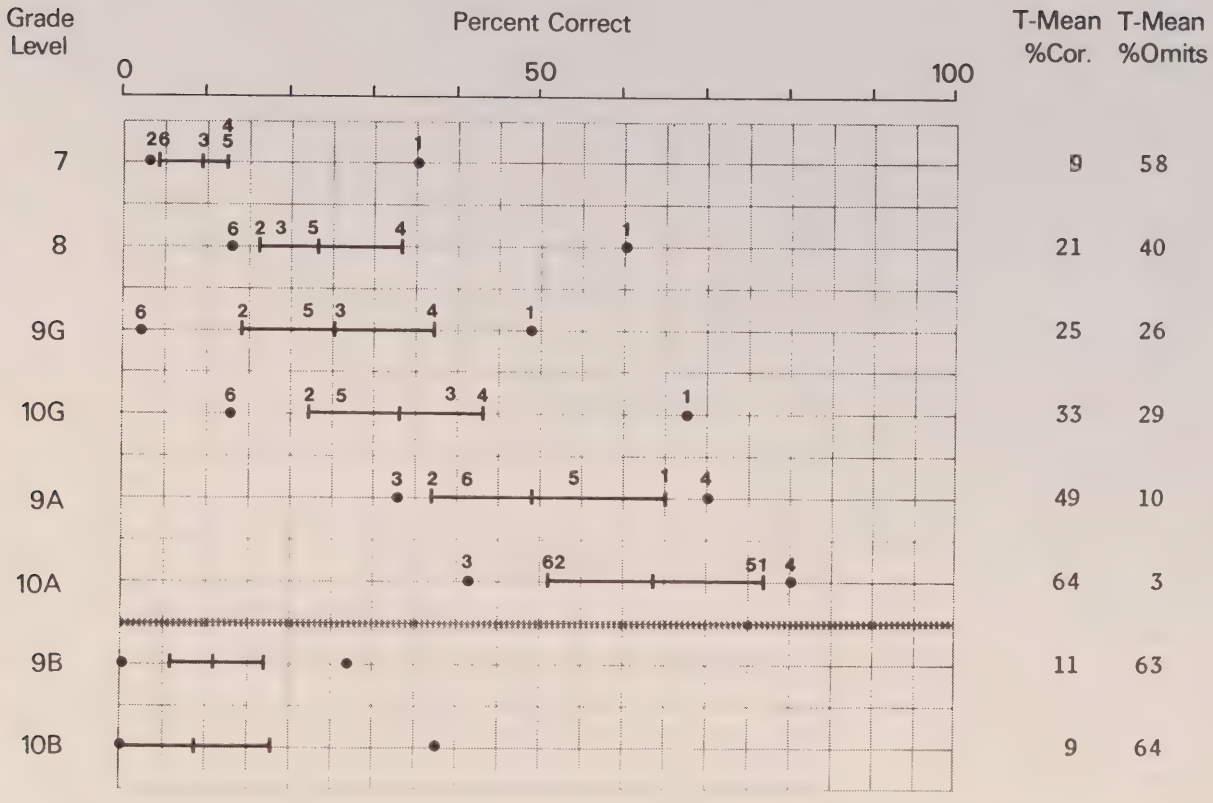
(5) Using the variable 'y', write an expression which means: "a number added to 33, then divided by 8"?

(6) Using the variable m, write an expression which means: "a number diminished by 20, then divided by 27"?

ALGEBRA--WORKING WITH FORMULAS

Objective: Converting Word Phrases to Algebraic Phrases

(Not in published Pool)



- student performance, while improving across the grades, is disappointing. This may be due to reading difficulties.
- a large number of students did not attempt the instruments even though the topic is core in Grade 7.
- this is an important topic which should be given increased emphasis.

ALGEBRA--WORKING WITH FORMULAS

Objective: Evaluating Expressions

(Not in published Pool)

Instruments Included in the Field Trial:

(1) If $f = -2$, then $3f + 3 + 5f$ =

(2) If $z = -2$, then $\frac{9z + 24}{2}$ =

(3) If $c = 8$, then $\frac{-24}{c} + 4 + c$ =

(4) If $q = -4$, then $\frac{10q}{2} + 8$ =

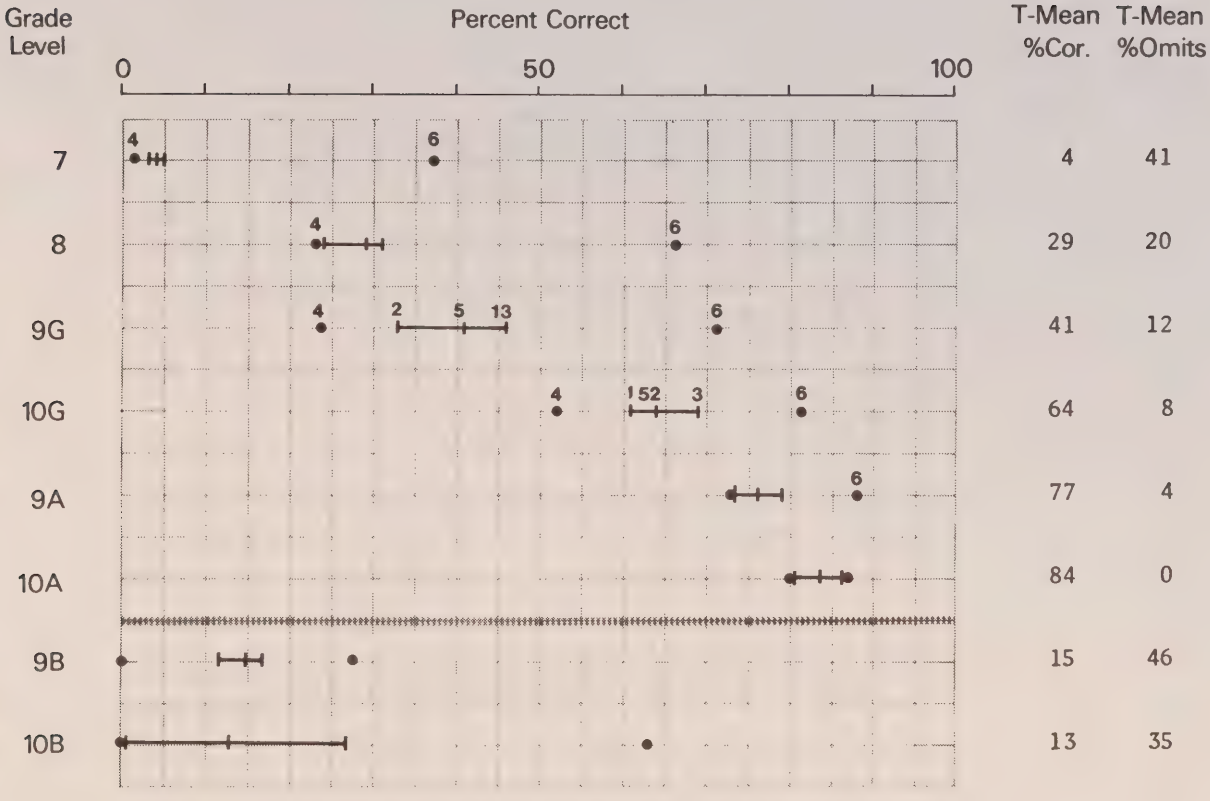
(5) If $m = -5$, then $10m + 3m$ =

(6) If $b = 9$, then $\frac{27}{b} + 9 + b$ =

ALGEBRA--WORKING WITH FORMULAS

Objective: Evaluating Expressions

(Not in published Pool)



- the range of performance on the instruments is consistent with operations with integers not being core in the Ministry guideline in for Grade 7 or for Grade 9 or 10 Basic.
- given where this topic occurs as core in the guidelines, the improvement in performance is very good.

ALGEBRA--WORKING WITH FORMULAS

Objective: Substituting In a Formula to Find Any Specific Variable

(N 012 200; p. 256)

Instruments Included in the Field Trial:

(1) Use $P = 2(L + w)$ to find w, if P=24, and L = 1

w =

(2) Use $A = \frac{bh}{2}$ to find h, if A=24, b=7

h =

(3) Use $A = \frac{h(b + a)}{2}$ to find b, if A=8, h=3, a=1

b =

(4) Use $V = Lwh$ to find w, if V=90, L=5 and h = 5

w =

(5) Use $A = Lw$ to find w, if A=100, L=8

w =

(6) Use $A = \frac{h(b + a)}{2}$ to find a, if A=32, h=3, b=1

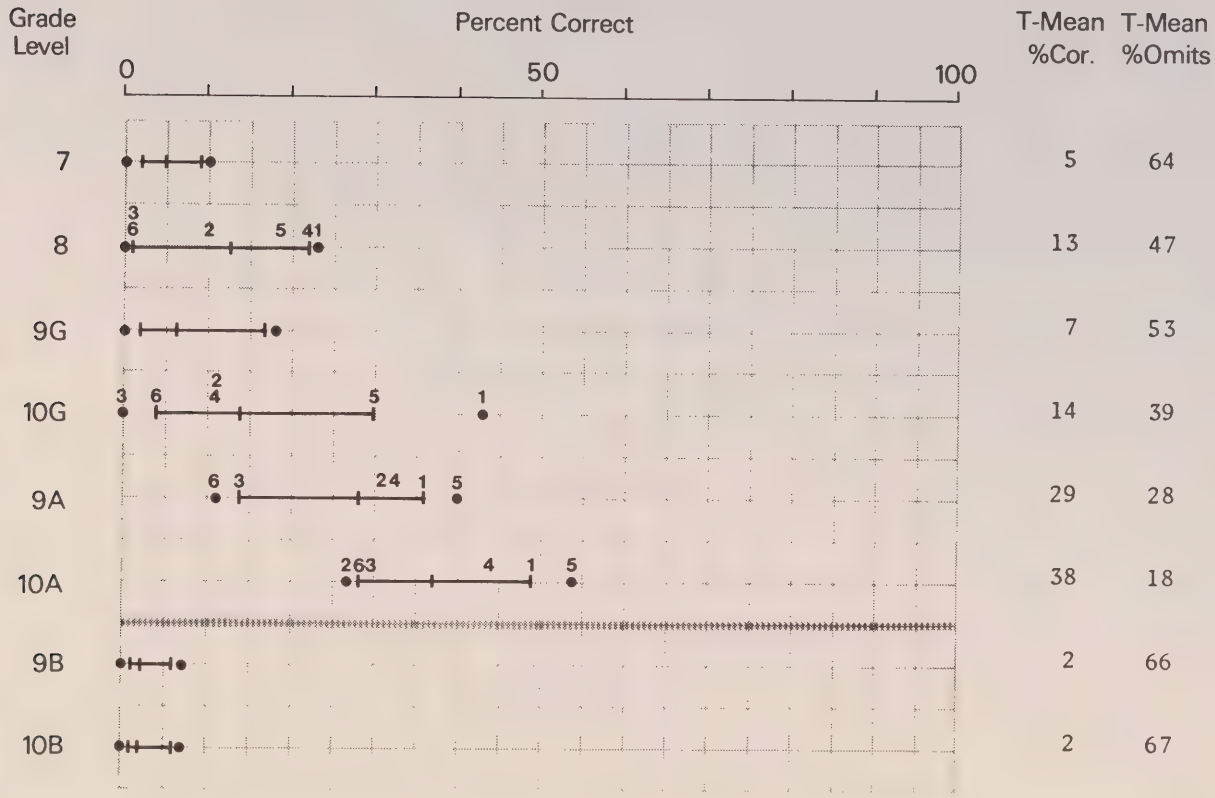
a =

ALGEBRA--WORKING WITH FORMULAS

Objective: Substituting In a Formula to Find Any Specific Variable

(N 012 200; p. 256)

Range of Correct Responses to the Six Instruments, by Grade:



- student performance illustrates their long-standing difficulty with this topic.
- according to the guidelines, it is highly unlikely that questions such as the most difficult would be encountered before Grade 10. This is confirmed by the number of omits.

ALGEBRA--WORKING WITH FORMULAS

Objective: Removing Brackets and Collecting Like Terms (Negative Coefficients)

(A 008 300; p. 235)

Instruments Included in the Field Trial:

(1) Simplify:

$-9(8a - 1) - 4 =$

(2) Simplify:

$4 - 9(6m - 8) =$

(3) Simplify:

$-5(9a - 1) + 6a =$

(4) Simplify:

$10t - 8(t + 3) =$

(5) Simplify:

$-8(6d + 8) =$

(6) Simplify:

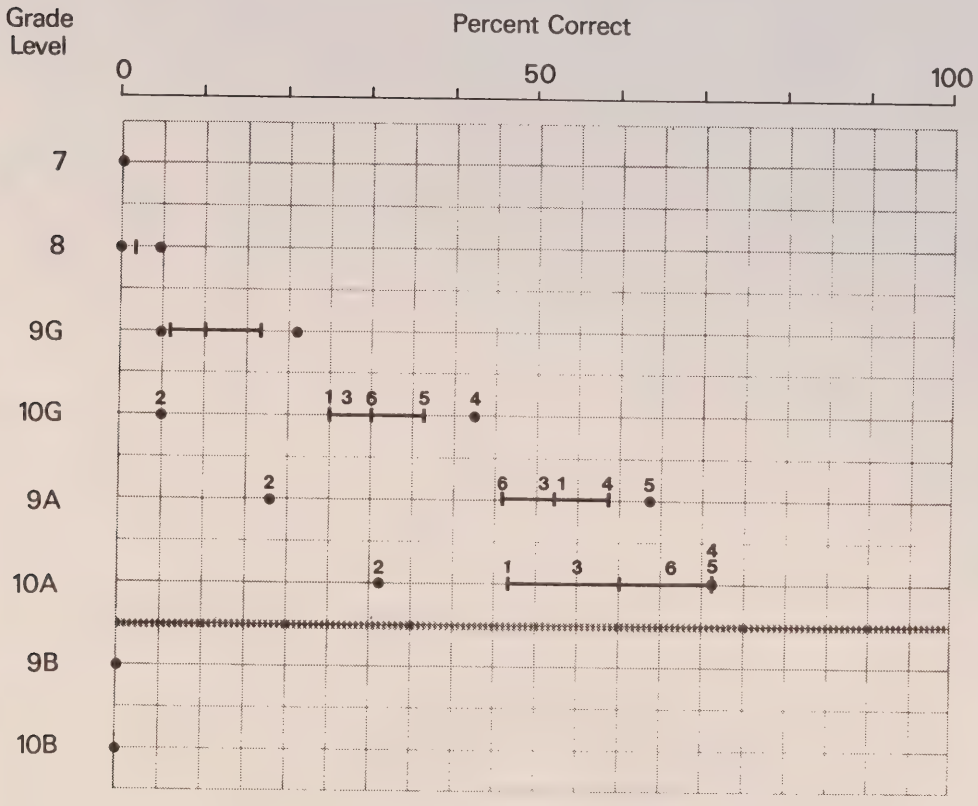
$-2(8m + 3) - 5(5m - 1) =$

ALGEBRA--WORKING WITH FORMULAS

Objective: Removing Brackets and Collecting Like Terms (Negative Coefficients)

(A 008 300; p. 235)

Range of Correct Responses to the Six Instruments, by Grade:

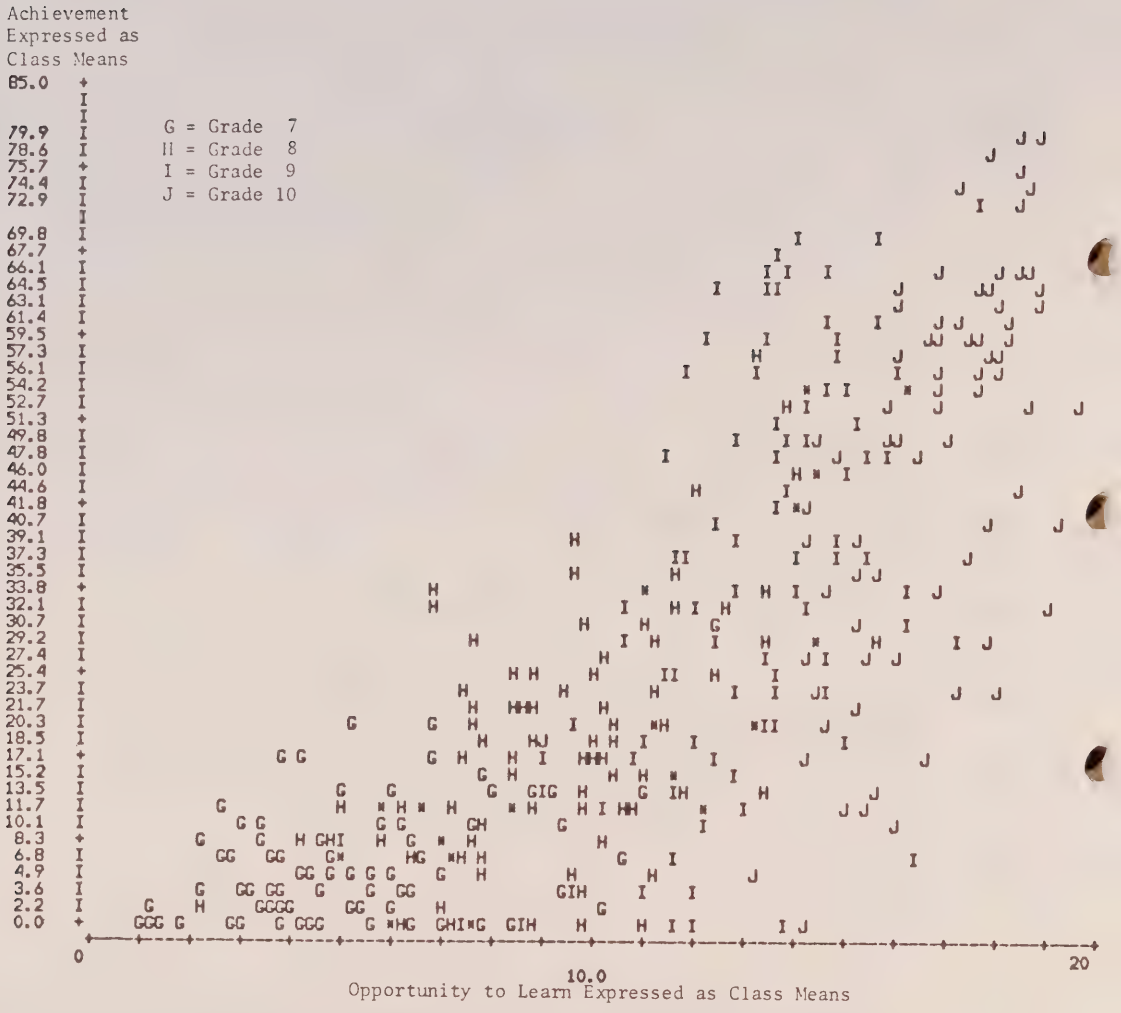


Grade Level	T-Mean %Cor.	T-Mean %Omits
7	0	75
8	1	57
9G	11	20
10G	30	16
9A	53	4
10A	60	2
9B	0	64
10B	0	48

- student performance and the number of omits is not surprising because this topic is not part of the curriculum for Gr. 7, 8, 9 Basic or 10 Basic.
- results indicate that where this skill is needed in Gr. 11 and 12, it should be reviewed and practised then.

ALGEBRA--WORKING WITH FORMULAS

Relationship Between Achievement and Opportunity to Learn:



ALGEBRA--WORKING WITH FORMULAS

Terminal Objectivies in Topic Group

Converting Word Phrases to Algebraic Phrases (Not in published Pool)

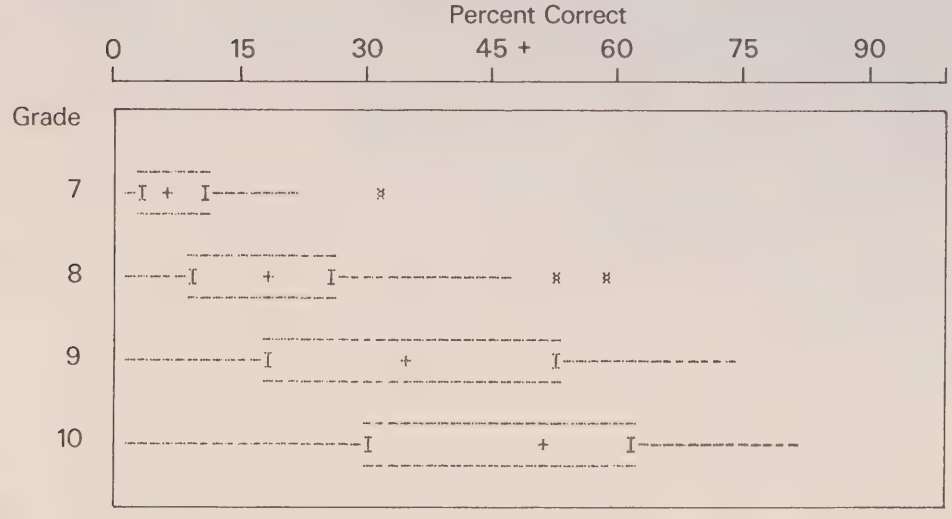
Evaluating Expressions (Not in published Pool)

Substituting In a Formula to Find Any Specific Variable (N 012 200; p. 256)

Removing Brackets and Collecting Like Terms (Negative Coefficients)

(A 008 300; p. 235)

Range of Correct Responses to Topic Group, by Grade:



Statistical Summary

Grade Level	No. of Classes	Grade Mean	Grade St. Dev.
7	86	6.2	6.1
8	93	18.3	12.0
9	101	34.2	20.9
10	85	45.5	19.9

Objective: Three-Step Equations/Multi-Step Equations (A 009 300; p. 242/ 400; p. 245)

Instruments Included in the Field Trial:

(1) $\frac{-9a}{3} + 8 = 5$ a =

(2) $-3(a + 8) - 10 = -43$ a =

(3) $\frac{2(57 + 6s)}{3} = 2$ s =

(4) $\frac{2a + 66}{8} = 8$ a =

(5) $\frac{14(-9a - 20)}{4} = -7$ a =

(6) $\frac{-6a}{10} + 5 = 8$ a =

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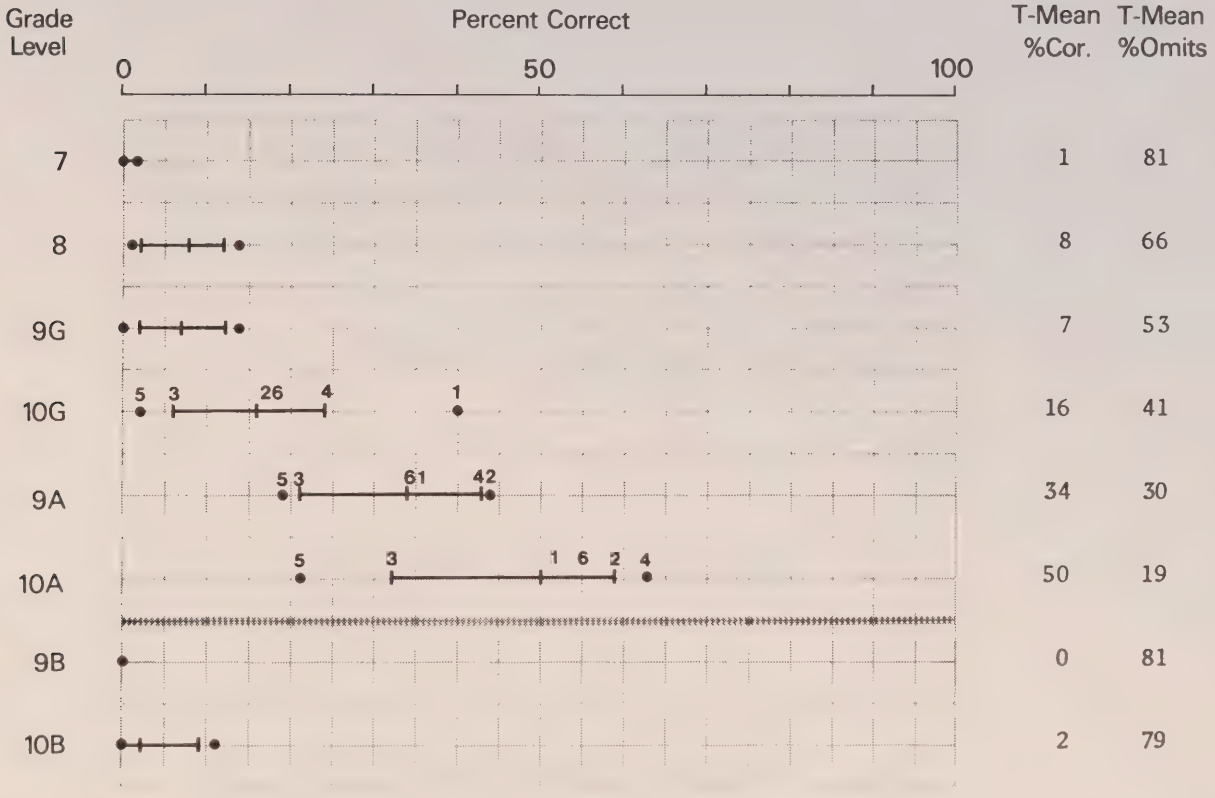
ALGEBRA--EQUATIONS AND INEQUALITIES

Objective: Three-Step Equations/Multi-Step Equations (A 009 300; p. 242/ 400; p. 245)

Range of Correct Responses to the Six Instruments, by Grade:



- this topic is not part of the Gr. 7 or Gr. 8 program.
- a surprisingly large number of Grade 10 Advanced students omitted these items.
- results indicate that where this skill is needed in Grade 11 and 12 it should be reviewed and practised then.

ALGEBRA--EQUATIONS AND INEQUALITIES

Objective: Solving Equations, Simplifying on Each Side

(Not in published Pool)

Instruments Included in the Field Trial:

(1) $-2(n - 7) - 3(n + 7) = 5(n + 11)$

$n =$

(2) $-(b + 15) + 4(b + 10) = -2(b + 10)$

$b =$

(3) $4(m + 13) + 10(m + 7) = 11(m - 12)$

$m =$

(4) $8(b + 8) - (b - 12) = 10(b - 4)$

$b =$

(5) $-6(m + 15) - 3(m + 6) = 9(m - 2)$

$m =$

(6) $7(x + 6) - 4(x + 9) = 6(x + 7)$

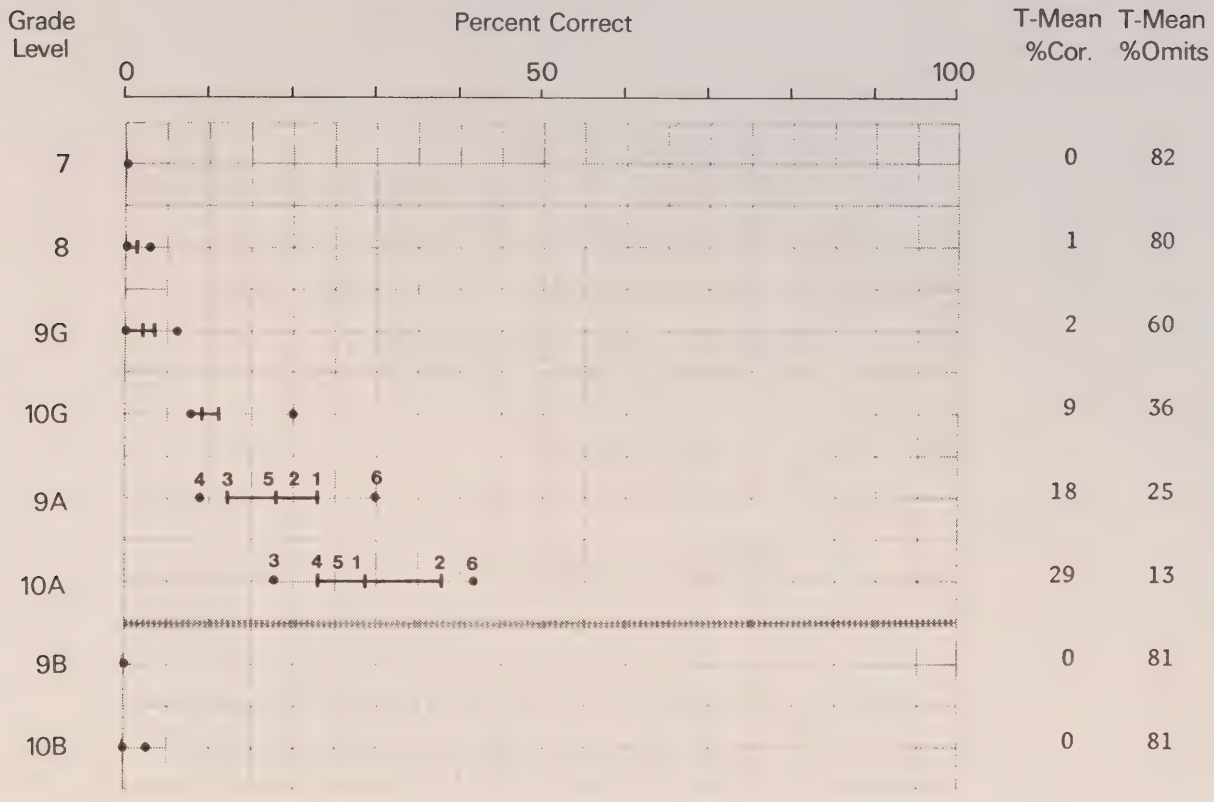
$x =$

ALGEBRA--EQUATIONS AND INEQUALITIES

Objective: Solving Equations, Simplifying on Each Side

(Not in published Pool)

Range of Correct Responses to the Six Instruments, by Grade:



- this skill is taught only in Gr. 9 and 10 Advanced classes so the poor results are not surprising.

Objective: Solving Equations With the Variable Appearing on Both Sides

(A 011 200; p. 252)

Instruments Included in the Field Trial:

(1) $\frac{3(3n - 2)}{2} = \frac{2(4n + 4)}{3}$ n =

(2) $\frac{4(2b - 2)}{3} + \frac{4}{2} = \frac{4(b - 4)}{2}$ b =

(3) $\frac{4b - 1}{3} + \frac{2(b + 4)}{3} = \frac{2(b - 4)}{4}$ b =

(4) $\frac{3(4n - 1)}{2} = \frac{4(2n - 1)}{3}$ n =

(5) $\frac{3(2b - 4)}{2} - \frac{1}{4} = \frac{2(3b - 4)}{4}$ b =

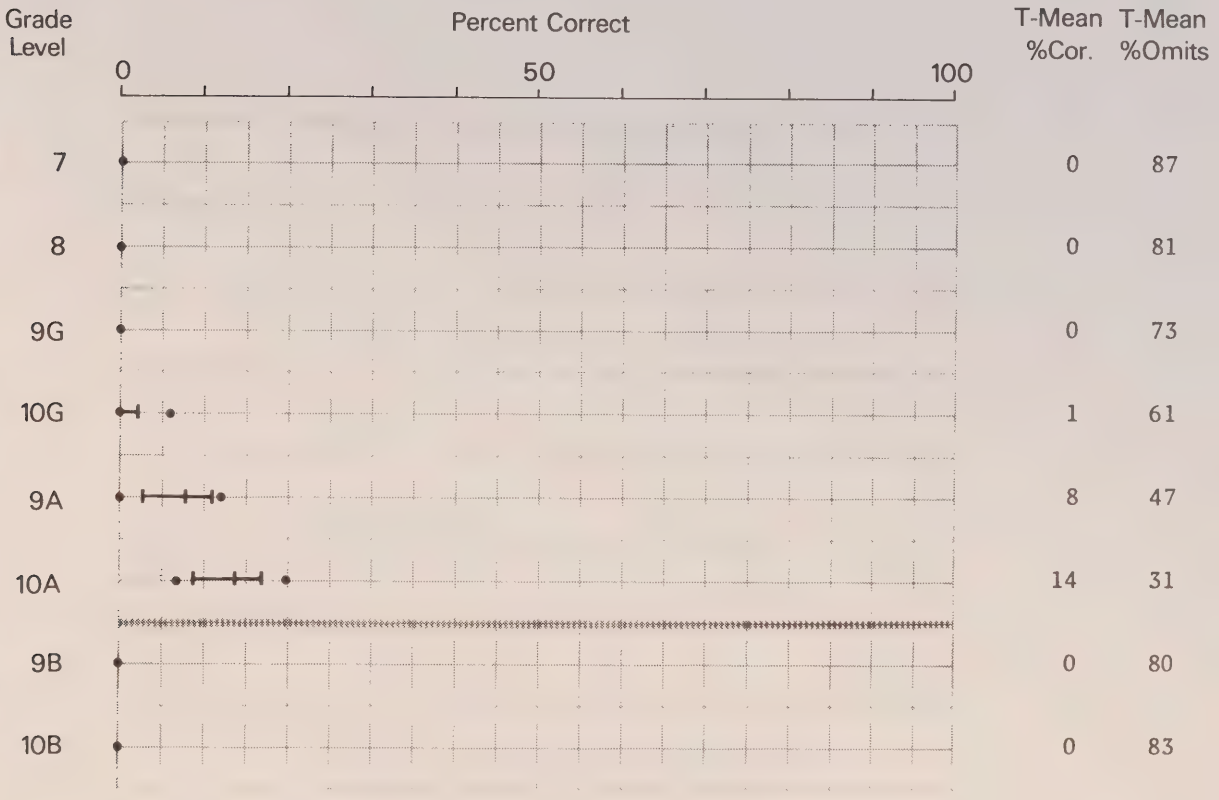
(6) $\frac{4b - 2}{2} + \frac{3(b + 3)}{4} = \frac{3(b - 4)}{4}$, b =

ALGEBRA--EQUATIONS AND INEQUALITIES

Objective: Solving Equations With the Variable Appearing on Both Sides

(A 011 200; p. 252)

Range of Correct Responses to the Six Instruments, by Grade:



- this skill is taught only in Gr. 9 and 10 Advanced classes.
- there were a high number of omits for students in these classes.
- it is obvious that more time and stress on this topic is needed at the appropriate level.
- users are advised to employ instruments of a less complex form unless scoring provides for partial credit.

Objective: Solving a Formula for Any Specific Variable (A 012 300; p. 258)

Instruments Included in the Field Trial:

(1) Rearrange the formula $A = \frac{bh}{2}$ as $b =$

(2) Rearrange the formula $v = u + at$ as $u =$

(3) Rearrange the formula $A = \frac{h(b + a)}{2}$ as $h =$

(4) Rearrange the formula $S = \frac{(a + L)}{2}$ as $L =$

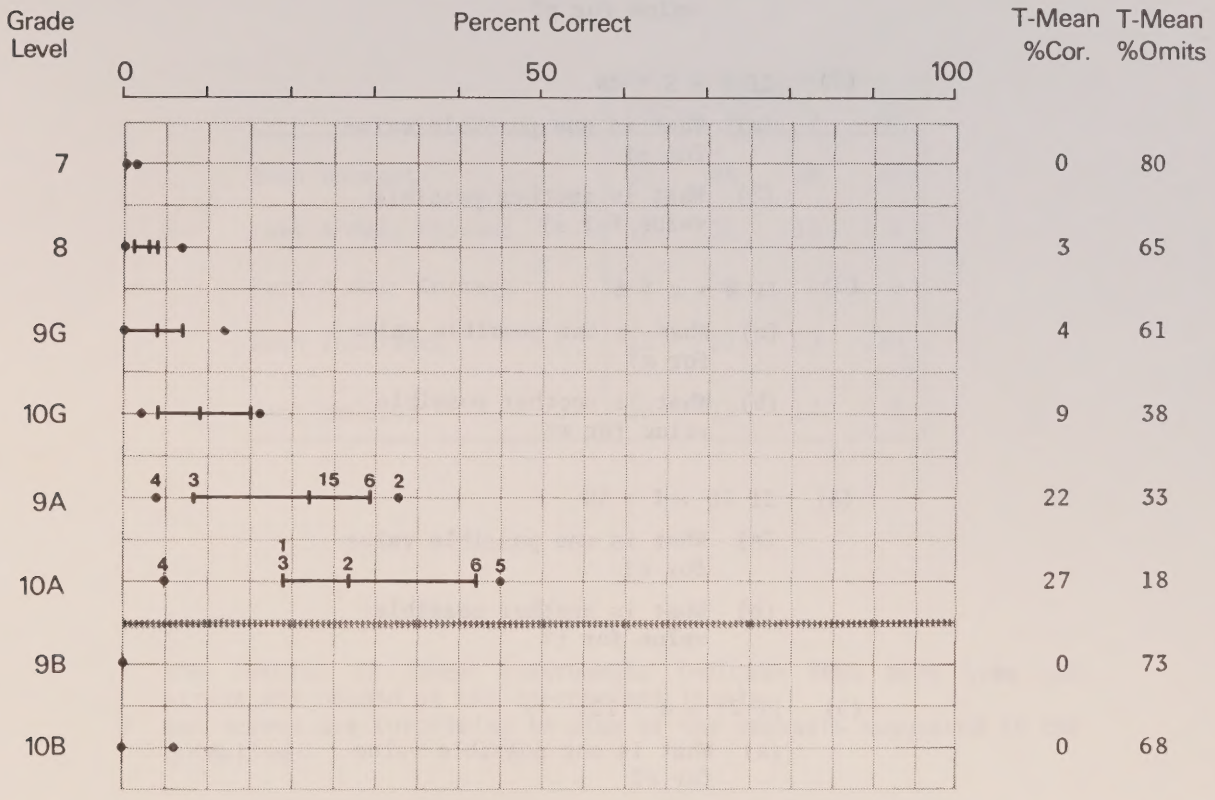
(5) Rearrange the formula $V = Lwh$ as $w =$

(6) Rearrange the formula $d = rt$ as $r =$

ALGEBRA--EQUATIONS AND INEQUALITIES

Objective: Solving a Formula for Any Specific Variable (A 012 300; p. 258)

Range of Correct Responses to the Six Instruments, by Grade:



- this skill is taught only in Gr. 9 and 10 Advanced classes.
- there were a high number of omits for students in these classes.
- it is obvious that more time and stress on this topic is needed at the appropriate level.

Objective: Solving Inequalities (A 005 300; p. 215)

Instruments Included in the Field Trial:

(1) If $8 + 2x < 13$

(a) What is one possible value for x?

(b) What is another possible value for x?

(2) If $s + 5 < 14$

(a) What is one possible value for s?

(b) What is another possible value for s?

(3) If $6 + x > 8$

(a) What is one possible value for x?

(b) What is another possible value for x?

(4) If $2t + 1 > 10$

(a) What is one possible value for t?

(b) What is another possible value for t?

(5) If $2t + 6 > 13$

(a) What is one possible value for t?

(b) What is another possible value for t?

(6) If $3 + 2n < 14$

(a) What is one possible for n?

(b) What is another possible value for n?

ALGEBRA--EQUATIONS AND INEQUALITIES

Objective: Solving Inequalities (A 005 300; p. 215)

Responses to Two-Part Instruments, by Grade:

	Grade			
	7	8	9	10
Both Correct	32	48	49	57
Part A Only Correct	12	11	10	9
Part B Only Correct	0	0	0	0
Both Incorrect	22	21	23	20
Both Omitted	30	15	14	9

- the results on these instruments indicate that more time and stress are needed at the appropriate levels.
- the scores are surprising in view of the emphasis suggested in the guideline.

ALGEBRA--EQUATIONS AND INEQUALITIES

Terminal Objectives in Topic Group

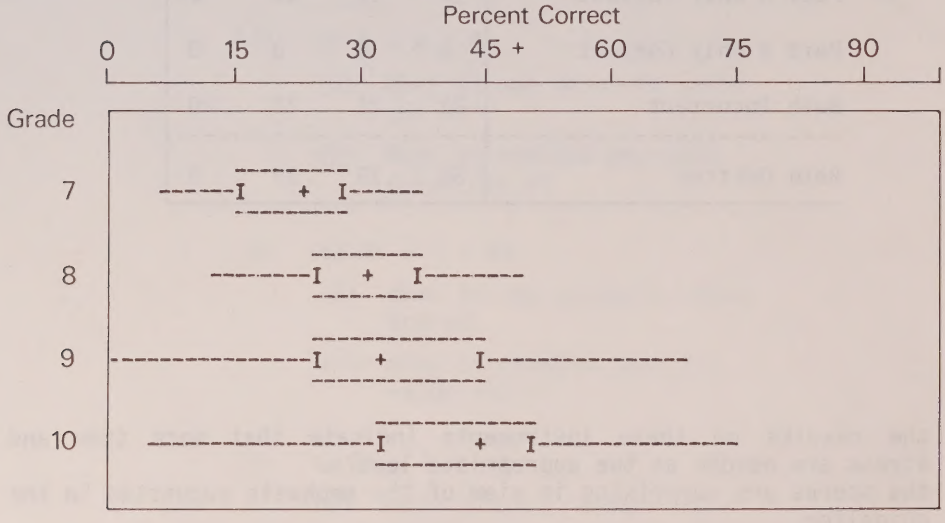
Three-Step Equations/Multi-Step Equations (A 009 300; p. 242/ 400; p. 245)

Solving Equations, Simplifying on Each Side (Not in published Pool)

Solving Equations With the Variable Appearing on Both Sides (A 011 200; p. 252)

Solving a Formula for Any Specific Variable (A 012 300; p. 258)

Range of Correct Responses to Topic Group, by Grade:



Statistical Summary

Grade Level	No. of Classes	Grade Mean	Grade St. Dev.
7	97	21.6	7.0
8	98	30.4	7.5
9	125	33.6	13.6
10	107	41.8	13.8